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Rate of convergence of attractors for singularly perturbed semilinear problems ☆

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A B S T R A C T

We exhibit a class of singularly perturbed parabolic problems which the asymptotic behavior can be described by a system of ordinary differential equation. We estimate the convergence of attractors in the Hausdorff metric by rate of convergence of resolvent operators. Application to spatial homogenization and large diffusion except in a neighborhood of a point will be considered.

1. Introduction

There are many parabolic problems whose asymptotic behavior is dictated by a system of Morse–Smale ordinary differential equations, for example, reaction diffusion equation where the diffusion coefficient become very large in all domain or reaction diffusion equation where the diffusion coefficient is very large except in a neighborhood of a finite number of points where it becomes small. These kind of problems was considered in the works [8,10–14], where well-posedness, functional setting and convergence of attractors was studied. In general it is considered a family of parabolic problems depending on a positive parameter ε and when ε converges to zero, it is obtained a limiting ordinary differential equation that contains all dynamic of the problem. Therefore the partial differential equation that generated an abstract parabolic problem in an infinite dimensional phase space can be considered in large time behavior as an ordinary differential equation in a finite dimensional space.

One of the most important part in these works is the study of an eigenvalue problem to determine that the first eigenvalues converge to matching eigenvalues of the limiting ODE, whereas all the others blow up, establishing the existence of a large gap between the eigenvalues as the parameter ε goes to zero. The large

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gap between the eigenvalues is known as gap condition. It is the main property that enable us construct an invariant manifold given as graph of certain Lipschitz map defined in a finite dimensional space that becomes flat, as the parameter varies. These manifolds are differentiable and since they are invariant we can restrict the flow to them and project the flow in a finite dimensional space. The theory of invariant manifold is well developed in many works, see [11,17,20,23].

The main property about Morse–Smale problems is structural stability under small C^1 perturbation, thus, it is interesting treat the problems described above as singular perturbation of a Morse–Smale limiting ODE, hence there is an isomorphism between the attractors in the sense that all equilibrium points and connection between them are preserved, see [6].

We are interested in knowing how fast the dynamic above approach each other, more precisely, we want to estimate the convergence of attractors when the parameter ε converges to zero. In this direction we follow the works [1,5,22], where a rate of convergence for the attractors was obtained assuming a rate of convergence for the resolvent operators. However the rates obtained in these works always show a loss with respect to rate of resolvent operators considered.

In this paper we will show that for a class of parabolic problem that can be regarded as an ordinary differential equation we have the optimal rate, in the sense that we do not have loss in the process to pass the convergence of the resolvent operators to convergence of attractors. Thus we improve for a class of singularly perturbed semilinear parabolic PDE's existing results on rate of convergence of attractors initially addressed by [5], where the rates obtained were not optimal. Even for the particular situation presented in [22] where the system is Morse–Smale the rate is not optimal. The main reason for estimating the convergence of attractors in this way is because we can actually ensure that the dynamics of the perturbed problems and unperturbed problem approach each other in the same speed as the equilibrium points, eigenfunctions, eigenvalues, spectral projections and invariant manifolds. Since the attractor is an invariant compact set that contains all the dynamics of the equation, equilibria points and connections between them, it is important that the rate of convergence of the attractors is the same as, for example, the rate of convergence of equilibrium points. We also improve for our class of equations the question originated in [1] where the rate of convergence of local unstable manifolds is a lot better than the rate of convergence of global attractors.

This paper is organized as follows. In the Section 2 we developed an abstract functional framework to treat the parabolic problem whose asymptotic behavior is described by a system of ordinary differential equation, we introduce the usual notation and define some notions of theory of attractor for semigroups. In the Section 3 we introduce the important notion of compact convergence and prove the convergence of eigenvalues and eigenfunctions, we also obtain the gap condition and estimates a priori on linear semigroups that is essential to construct the invariant manifold. In the Sections 4, 5 and 6 we show that the rate of convergence of resolvent operators is the same rate of convergence of invariant manifolds and attractors, for that end, some aspects of Shadowing Theory will be presented. The Section 7 is devoted to comments about more general situations that can be considered. Finally, in the Sections 8 and 9 we consider the examples of spatial homogenization and large diffusion except in a neighborhood of a point where it becomes small.

2. Functional setting

In this section we introduce the general framework that we use to treat parabolic problems whose asymptotic behavior is dictated by a system of Morse–Smale ordinary differential equations. Some notion about the theory of attractors for semigroups is presented. We consider here only positive self-adjoint operator with compact resolvent. More general situation will be addressed in the Section 7.

Let X_0 be a finite dimensional Hilbert space with $\dim(X_0) = n$, for some positive integer n, and let $A_0 : X_0 \to X_0$ be an invertible bounded linear operator whose spectrum set $\sigma(A_0)$ is given by

$$\sigma(A_0) = \{\lambda_1^0 < \lambda_2^0 < \cdots < \lambda_n^0\}, \quad 0 < \lambda_1^0.$$