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Uniqueness in the determination of unknown coefficients of an Euler–Bernoulli beam equation with observation in an arbitrary small interval of time

Alexandre Kawano

Escola Politecnica, University of Sao Paulo, Brazil

A R T I C L E I N F O

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A B S T R A C T

In the present article, we prove an uniqueness result for the non-linear problem of identifying the flexural stiffness and density of an Euler–Bernoulli beam using observation of boundary measurements. Here we show that the knowledge of the displacement and slope of at the free extremity of a clamped-free vibrating beam, for an arbitrary small interval of time, leads to the uniqueness in the identification of its rigidity and density. This result extends previous works that showed that the identification was possible when the observation was collected in an unbounded interval of time, or when the original dynamic problem for the Euler–Bernoulli beam was linearised.

1. Introduction

The main physical application behind the motivation of the study, mathematical in character, reported in the present article is the identification of distributed parameters over structural elements. The pursuit of controlling or keeping safe structural systems continues to be an important theme of modern research. Structures change their properties or deteriorate by several mechanisms, such as corrosion, fatigue or even accidental damage and therefore, it is vital to detect imperfections in structural elements timely by non-destructive methods. Just to cite some relevant works concerning research in the determination of defects in beams, which constitute one of the main structural elements, we mention [2,6,7,13,14,18,24,26,27].

Inverse problems concerning the determination of extensive properties such as the rigidity or density distributions along the beam from the knowledge of spectral information were performed by [3], Mclaughlin [19], Gladwell [9] and Morassi [25] among others. Barcilon [3] showed that uniqueness in the identification of the rigidity and mass distribution of a beam is possible when three interlacing sequences of eigenvalues corresponding to three different boundary conditions are given. He also showed that the knowledge of

E-mail address: akawano@usp.br.

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the impulse response, for all times, measured at the free extremity of a clamped beam also leads to the uniqueness in the recovery of the properties of the beam. The same result for the impulse response was obtained by Chang and Guo [7] by using a different method. Mclaughlin wrote a series of articles [19–23] about the recovery of coefficients of fourth order operators, including the Euler–Bernoulli beam. Calling $\rho(x)$ the density and $EI(x)$ the rigidity functions respectively, and defining $a^2 = \frac{\rho}{EI}$ and $c^2 = (EI)^{3/4}\rho^{1/4}$, she proved that the knowledge of $a(0) = 1$, $c(0) = 1$, the set of eigenvalues $0 < \lambda_1 < \lambda_2 < \dots$, plus the knowledge of the sequences $(S_n(0))_{n\in\mathbb{N}}$ and $(S'_n(0))_{n\in\mathbb{N}}$ where S_n, $n \in \mathbb{N}$, are the normalised eigenfunctions of the Euler–Bernoulli operator (normalisation by $\int_0^1 a^2c^2S_n^2\,\mathrm{d}x = 1$), $\forall n \in \mathbb{N}$, are enough to guarantee uniqueness in the identification of ρ and EI, provided an integral equation relating the original problem to another with known coefficients is satisfied.

We remark that the problem is physically, as well as mathematically challenging, because it is known that there are beams with different physical properties that possess the same set of eigenvalues. This fact was pointed out by Barcilon [3], Gottlieb [12], Caudill, Lester, Perry & Schueller [5] and Gladwell & Morassi [11].

In the present article, we intend to show a proof that the observation of the displacement and slope of the vibrating beam at the free extremity of a clamped-free Euler–Bernoulli beam, *for an arbitrary small interval of time*, leads to the uniqueness in the identification of the rigidity $EI(x)$ and density $\rho(x)$ of it. This is in contrast with all above mentioned articles, which require an unbounded interval of time to make the observation.

Here we are not going to linearise this inverse problem.

Consider the following problem

$$\begin{cases} \rho\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = \delta\otimes f, & (t,x)\in[0,T_0[\times[0,L[,\\ w(t,x) = 0, & \forall t\le 0, \forall x\in[0,L],\\ \frac{\partial w}{\partial t}(t,x) = 0, & \forall t< 0, \forall x\in[0,L],\\ \frac{\partial^2 w}{\partial x^2}(t,0) = \frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)(t,0) = 0, & \forall t\in]0,T_0[,\\ w(t,L) = \frac{\partial w}{\partial x}(t,L) = 0, & \forall t\in[0,T_0[, \end{cases} \tag{1}$$

where $\rho \in \mathcal{C}^4([0,L])$, $\rho > 0$, is the mass density per unit length, $EI \in \mathcal{C}^4([0,L])$, $EI > 0$, is the rigidity, δ is the Dirac's Delta distribution, $f \in \mathrm{H}^{-1}(]0,L[)$ the spatial loading imposed to the beam and w is the displacement. $\delta\otimes f$ denotes simply the tensor product of δ and f, following the notation contained in, for instance, [29].

Physically, the boundary conditions that appear in (1) correspond to the free-clamped beam configuration. The clamped boundary condition at $x = L$ is clear. The ones at $x = 0$ mean that there are no forces nor bending moments acting at the tip $x = 0$.

We will prove that when $f = \delta$, the rigidity $EI \in \mathcal{C}^4([0,L])$ and the density $\rho \in \mathcal{C}^4([0,L])$, both positive functions, that appear in (1) can be identified uniquely given the knowledge of the set

$$\Gamma = \{(w(t,0), \partial_x w(t,0), t) \,:\, t\in\,]0,T[\}\,, \tag{2}$$

where $0 < T < T_0$ can be arbitrarily small.

The proof consists of an application of a result for almost periodic distributions found in [16], and an application of a result for the Euler-Beam equation due to Mclaughlin [23] and Gladwell [10].

2. Direct problem

Since the operator in problem (1) is of forth order, and we use below the Galerkin Method, the natural space to use here is the Sobolev space $\mathrm{H}^2(]0,L[)$. However, since problem (1) poses Dirichlet boundary