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Energy decay rates for the elasticity system with structural damping in the Fourier space ☆

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ABSTRACT

This paper deals with a Cauchy problem for the elasticity system with a structural damping. The decay rates of the total energy of the system are presented by applying the Fourier transform and multiplier in the Fourier space. Furthermore, almost optimal decay estimates of the total energy are obtained based on the Haraux–Komornik inequality.

1. Introduction

We consider the Cauchy problem for the following system in $\mathbb{R}^n$ $(n > 1)$

$$\begin{cases} u_{tt}(t,x) + Au(t,x) + A^\theta u_t(t,x) = 0, & (t,x) \in (0,\infty) \times \mathbb{R}^n, \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in \mathbb{R}^n, \end{cases} \tag{1.1}$$

where the operator A is denoted as

$$Au := -\mu\Delta u - (\lambda + \mu)\nabla \text{div} u,$$

with λ and μ are Lamé constants satisfying

$$\mu > 0, \ n(\lambda + \mu) + \mu > 0. \tag{1.2}$$

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The vector displacement $u = u(t,x)$ represents $u = (u^1, u^2, \dots, u^n)^T$ with $u^i = u^i(t,x)$ for each $i = 1, \dots, n$, and the initial data are denoted as $u_j(x) = (u_j^1(x), u_j^2(x), \dots, u_j^n(x))^T$ for $j = 0, 1$.

The system (1.1), which interpolates between the weak damping case $\theta = 0$ and the strong damping case $\theta = 1$, is referred to as the structural damped elasticity system (see [2,4]).

For the single structural damped wave equation

$$\begin{cases} u_{tt}(t,x) - \Delta u(t,x) + (-\Delta)^\theta u_t(t,x) = 0, & (t,x) \in (0,\infty) \times \mathbb{R}^n, \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in \mathbb{R}^n, \end{cases} \tag{1.3}$$

we first remind the work due to Karch [6], in which Karch investigated the asymptotic self-similarity in the large time of solution to (1.3) with $\theta \in (0, 1/2)$ through a multiple of the Gauss–Weierstrass kernel. Recently, Ikehata–Natsume [4] studied the diffusive property of (1.3) in the case of $\theta \in [1/2, 1]$, they also obtained decay estimates for the total energy of solutions with $\theta \in [0, 1]$. Through the combination of the energy method in the Fourier space and the Haraux–Komornik inequality, Charão–da Luz–Ikehata [2] re-studied problem (1.3) and derived more sharp energy estimates in the case of $\theta \in (0, 1/2]$.

For the system of elastic wave with the structural damping, we mention the work of Ikehata–da Luz–Charão [3], in which the Cauchy problem of the following system

$$u_{tt}(t,x) - a^2 \Delta u(t,x) - (b^2 - a^2)\nabla \mathrm{div} u(t,x) + (-\Delta)^\theta u_t(t,x) = 0 \tag{1.4}$$

was discussed. After applying an improved energy method in the Fourier space, they obtained the decay rates for the total energy with $\theta \in [0, 1]$ and the estimates for L^2-norm of solutions to (1.4).

We note that all the papers mentioned above deal with only the case that the structural damping is $(-\Delta)^\theta u_t$. In this paper, we are concerned with the energy decay rates of solutions to problem (1.1) with $\theta \in [0, 1]$. Throughout this paper we use the notation $\|\cdot\|_p$ for the $(L^p(\mathbb{R}^n))^n$ norms of functions. Particularly, $\|\cdot\|$ denotes the $(L^2(\mathbb{R}^n))^n$ norm for simplicity. Let s be a nonnegative number, then $H^s(\mathbb{R}^n)$ denotes the Sobolev Space. The Fourier transform of the function $v \in (L^2(\mathbb{R}^n))^n$ is denoted as $\hat{v} = (\hat{v}^1, \hat{v}^2, \dots, \hat{v}^n)^T$, where

$$\hat{v}^j = \hat{v}^j(t,\xi) = \mathcal{F} v^j(\xi) = \int_{\mathbb{R}^n} \mathrm{e}^{-ix\cdot\xi} v^j(x) dx, \quad \xi = (\xi_1, \xi_2, \dots, \xi_n)^T \in \mathbb{R}^n_\xi, \quad j = 1, 2, \dots, n,$$

with its inverse $\mathcal{F}^{-1}$.

The total energy associated with (1.1) is defined by

$$E_u(t) = \frac{1}{2} \int_{\mathbb{R}^n} (|u_t|^2 + \mu |\nabla u|^2 + (\lambda + \mu)|\mathrm{div} u|^2) dx.$$

From (1.1), we have (at least formally) the following identity

$$E_u(t) + \int_0^t \|A^{\theta/2} u_t(s)\|^2 ds = E_u(0).$$

Though the above identity indicates that the function $t \to E_u(t)$ is monotone decreasing, obtaining the decay rates of the total energy seems difficult. For one thing, it is difficult to give the explicit expression of solutions to (1.1) because of the cross derivatives in the system. For another, the structural operator A^θ is complicated and crucial in the estimates of $E_u(t)$.

By following from the equivalent definition of operator A as

$$Av(x) := \mathcal{F}^{-1}((\mu|\xi|^2 I + (\lambda + \mu)\xi\xi^T)\hat{v})(x), \quad \xi = (\xi_1, \cdots, \xi_n)^T \in \mathbb{R}^n_\xi,$$

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