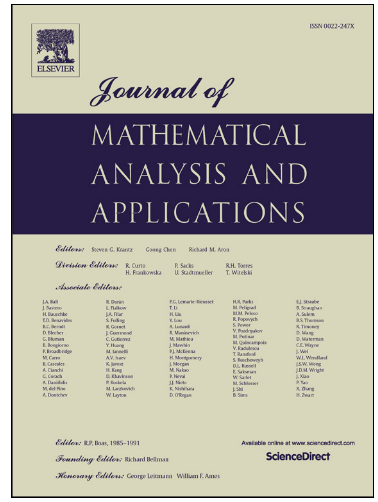

Existence and nonexistence results for critical biharmonic systems involving multiple singularities

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Existence and nonexistence results for critical biharmonic systems involving multiple singularities

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Abstract

In this paper, a system of biharmonic equations is investigated, which involves critical Sobolev nonlinearities and multiple singular points. By variational methods and analytical techniques, the best Soblev constant corresponding to the system is investigated, and the existence (nonexistence) of ground state solutions to the system is established.

Keywords: Biharmonic system; solution; critical nonlinearity; Rellich potential; variational method.

Mathematics Subject Classification 2010: 35B33, 35J35, 35J50, 35B25

1 Introduction

In this paper, we study the following system of elliptic equations:

$$\begin{cases} \Delta^2 u - \displaystyle\sum_{i=1}^{k} \frac{\lambda_i}{|x-a_i|^4} u = \big(|u|^q + |v|^q\big)^{\frac{2^*}{q}-1} |u|^{q-2} u, \\ \Delta^2 v - \displaystyle\sum_{i=1}^{k} \frac{\mu_i}{|x-b_i|^4} v = \big(|u|^q + |v|^q\big)^{\frac{2^*}{q}-1} |v|^{q-2} v, \\ u, v \in D^{2,2}(\mathbb{R}^N) \setminus \{(0,0)\}, \end{cases} \tag{1.1}$$

where the parameters satisfy the following condition:

$(\mathcal{H}_1)$ $N \geq 5,\ k \geq 2,\ k \in \mathbb{N},\ a_i, b_i \in \mathbb{R}^N,\ a_i \neq a_j,\ b_i \neq b_j,\ i,j = 1,2,\cdots,k,\ i \neq j,$
$1 < q \leq 2^*,\ \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_k < \bar{\lambda},\ \ \mu_1 \leq \mu_2 \leq \cdots \leq \mu_k < \bar{\lambda}.$

Furthermore, $D^{2,2}(\mathbb{R}^N) =: D$ is the completion of $C_0^\infty(\mathbb{R}^N)$ with respect to the norm $(\int_{\mathbb{R}^N} |\Delta \cdot|^2 \,\mathrm{d}x)^{1/2}$, $\bar{\lambda} := (\frac{N(N-4)}{4})^2$ is the best Rellich constant and $2^* := \frac{2N}{N-4}$ is the critical Sobolev exponent.

Define the energy functional corresponding to (1.1):

$$I(u,v) = \frac{1}{2}\mathcal{Q}(u,v) - \frac{1}{2^*}\int_{\mathbb{R}^N} \big(|u|^q + |v|^q\big)^{\frac{2^*}{q}} \mathrm{d}x, \quad (u,v) \in D \times D.$$

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