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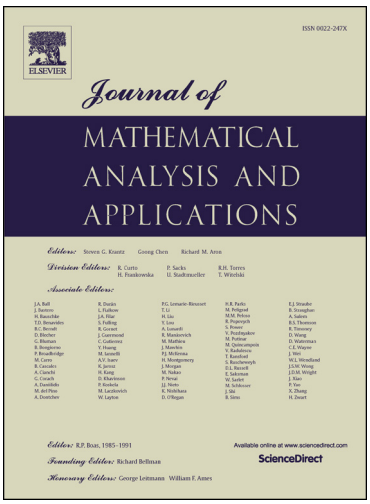

Monotone and convex restrictions of continuous functions

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Monotone and convex restrictions of continuous functions

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Dedicated to Jean-Pierre Kahane on the occasion of his 90^{ieth} *birthday*

Abstract

Suppose that f belongs to a suitably defined complete metric space $\mathcal{C}^\alpha$ of Hölder α-functions defined on $[0,1]$. We are interested in whether one can find large (in the sense of Hausdorff, or lower/upper Minkowski dimension) sets $A \subset [0,1]$ such that $f|_A$ is monotone, or convex/concave. Some of our results are about generic functions in $\mathcal{C}^\alpha$ like the following one: we prove that for a generic $f \in C_1^\alpha[0,1]$, $0 < \alpha < 2$ for any $A \subset [0,1]$ such that $f|_A$ is convex, or concave we have $\dim_{\mathrm{H}} A \leq \underline{\dim}_{\mathrm{M}} A \leq \max\{0, \alpha - 1\}$. On the other hand we also have some results about all functions belonging to a certain space. For example the previous result is complemented by the following one: for $1 < \alpha \leq 2$ for any $f \in C^\alpha[0,1]$ there is always a set $A \subset [0,1]$ such that $\dim_{\mathrm{H}} A = \alpha - 1$ and $f|_A$ is convex, or concave on A.

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