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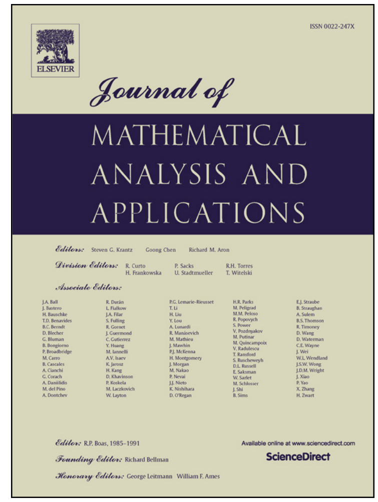

Distributionally chaotic systems of type 2 and rigidity

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DISTRIBUTIONALLY CHAOTIC SYSTEMS OF TYPE 2 AND RIGIDITY

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Abstract. In this paper we deal with uniformly rigid systems obtained by a method introduced by Katznelson and Weiss and show that such systems never contain DC2 pairs. On the other hand, we introduce a modification of this technique that leads to a uniformly rigid system with DC2 pairs. We also show that every dynamical system contains a pair of distinct points which is not DC2.

1. Introduction

Irrational rotations are examples of dynamical systems with very rigid structure of orbits, in the sense that all points return arbitrarily close their initial states simultaneously. This property is expressed formally in the following definition.

Definition 1.1. A dynamical system (X, f) is *uniformly rigid* if for every $\varepsilon > 0$ there is n such that $d(f^n(x), x) < \varepsilon$ for every $x \in X$.

In other words, a dynamical system (X, f) is *uniformly rigid* provided that $\liminf_{n\to\infty} \sup_{x\in X} d(f^n(x), x) = 0$. At first, it is not easy to decide if there are examples of uniformly rigid systems which are not equicontinuous, because it intuitively seems, that dynamics of such systems should be relatively simple. A big surprise was brought by a beautiful example by Glasner and Maon in [8] who proved that on every tori there are uniformly rigid minimal systems which at the same time are weakly mixing. A second general method of construction of transitive uniformly rigid systems originated from a work of Katznelson and Weiss [11]. In [1] Akin, Auslander and Berg extended this construction giving very precise description how to construct uniformly rigid dynamical systems following Katznelson and Weiss ideas. Among other things, they gave explicit conditions when this construction leads to an almost continuous dynamical system. It is also one of the first papers where it was proved that a transitive dynamical system is either sensitive or almost equicontinuous. It is also worth mentioning that examples constructed by application of methodology of [1] are either trivial or transitive but they are never minimal. In fact they are always transitive, proximal and uniformly rigid dynamical systems on a (possibly infinite-dimensional) continuum.

Recently, the construction of Katznelson and Weiss was modified in [4] to produce a weakly mixing, proximal and uniformly rigid dynamical system which reveals big potential behind the technique of Katznelson and Weiss.

As we can see, the dynamics of uniformly rigid systems can be much more complex than expected at first. The aim of this article is to present that a kind of

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