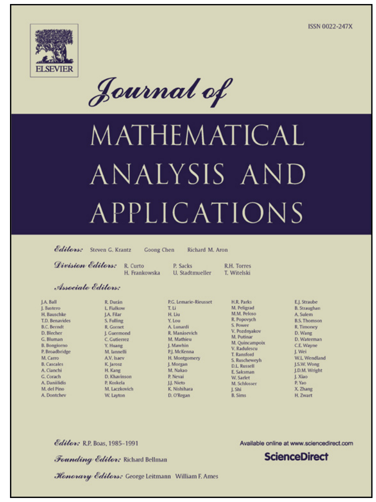
Accepted Manuscript



Commutants of separately radial Toeplitz operators in several variables

Trieu Le

PII: S0022-247X(17)30344-X
DOI: http://dx.doi.org/10.1016/j.jmaa.2017.03.088
Reference: YJMAA 21286

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 11 January 2017

COMMUTANTS OF SEPARATELY RADIAL TOEPLITZ OPERATORS IN SEVERAL VARIABLES

TRIEU LE

ABSTRACT. If φ is a bounded separately radial function on the unit ball, the Toeplitz operator T_φ is diagonalizable with respect to the standard orthogonal basis of monomials on the Bergman space. Given such a φ, we characterize bounded functions ψ for which T_ψ commutes with T_φ. Several examples are given to illustrate our results.

1. Introduction

Let $d \geq 1$ be a fixed integer. For $\mathbf{z} = (z_1, \ldots, z_d) \in \mathbb{C}^d$, we denote its Euclidean norm by $|\mathbf{z}| = \sqrt{|z_1|^2 + \cdots + |z_d|^2}$. We write $\mathbb{B}$ for the open unit ball consisting of all $\mathbf{z} \in \mathbb{C}^d$ with $|\mathbf{z}| < 1$. Let ν denote the Lebesgue measure on $\mathbb{B}$ normalized so that $\nu(\mathbb{B}) = 1$. The Bergman space A^2 consists of all holomorphic functions on $\mathbb{B}$ which are square integrable with respect to ν. Since A^2 is a closed subspace of the Hilbert space $L^2 = L^2(\mathbb{B}, \nu)$, there is an orthogonal projection P from L^2 onto A^2. For any bounded measurable function φ on the ball, the Toeplitz operator T_φ is defined by $T_\varphi h = P(\varphi h)$ for $h \in A^2$. It is immediate that T_φ is a bounded linear operator on A^2 with $\|T_\varphi\| \leq \|\varphi\|_\infty$. If φ is holomorphic on $\mathbb{B}$, then T_φ is the multiplication operator on A^2 with symbol φ.

The main goal of this paper is to study the *commuting problem* of Toeplitz operators on A^2: given a non-constant function φ, find the necessary and sufficient conditions on the function ψ such that $T_\varphi T_\psi = T_\psi T_\varphi$. The commuting problem for Toeplitz operators on the Hardy space of the unit disk was solved completely by Brown and Halmos in their seminal paper [8] back in the early sixties. Their result has motivated a vast literature on the studies of commuting Toeplitz operators acting on other Hilbert spaces of analytic functions: the Bergman space over the unit disk [2, 3, 10], the Hardy and Bergman spaces over the polydisk or the ball in higher dimensions [9, 11, 16, 17, 22] and the Fock spaces [6, 5, 1], just to list a few. The interested reader is referred to the above papers for more references. Quite often, an additional assumption on the function φ is imposed. In fact, even on the Bergman space over the unit disk, the general commuting problem remains open.

2010 *Mathematics Subject Classification.* Primary 47B35.

Key words and phrases. Bergman spaces; Toeplitz operators; commutants; separately radial functions.