Doctopic: Applied Mathematics ARTICLE IN PRESS YJMAA:21280

J. Math. Anal. Appl. ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Analysis of the non-reflecting boundary condition for the time-harmonic electromagnetic wave propagation in waveguides

Seungil Kim

Department of Mathematics and Research Institute for Basic Sciences, Kyung Hee University, Seoul 130-701, Republic of Korea

ARTICLE INFO

Article history:
Received 12 August 2016
Available online xxxx
Submitted by B. Kaltenbacher

Keywords:
Maxwell's equations
Non-reflecting boundary condition
Electric-to-magnetic operator
Waveguide

ABSTRACT

In this paper, we study the non-reflecting boundary condition for the time-harmonic Maxwell's equations in homogeneous waveguides with an inhomogeneous inclusion. We analyze a series representation of solutions to the Maxwell's equations satisfying the radiating condition at infinity, from which we develop the so-called electric-to-magnetic operator for the non-reflecting boundary condition. Infinite waveguides are truncated to a finite domain with a fictitious boundary on which the non-reflecting boundary condition based on the electric-to-magnetic operator is imposed. As the main goal, the well-posedness of the reduced problem will be proved. This study is important to develop numerical techniques of accurate absorbing boundary conditions for electromagnetic wave propagation in waveguides.

1. Introduction

In this paper, we will study the non-reflecting boundary condition for the electric fields $\boldsymbol{\mathcal{E}}$ satisfying the second-order Maxwell's equations in a waveguide

$$\begin{aligned} \nabla \times \nabla \times \boldsymbol{\mathcal{E}} - \omega^2 \epsilon \boldsymbol{\mathcal{E}} &= i\omega \boldsymbol{J} \text{ in } \Omega_\infty, \\ \boldsymbol{\nu} \times \boldsymbol{\mathcal{E}} &= 0 \text{ on } \partial\Omega_\infty, \end{aligned} \tag{1.1}$$

where Ω_∞ is a semi-infinite waveguide in $\mathbb{R}^3$ with Lipschitz boundary such that

$$\Omega_\infty \cap \{(x, y, z) \in \mathbb{R}^3 \;:\; z > -b\} = \Theta \times (-b, \infty)$$

and $\Omega_\infty \cap \{z < -b\}$ is bounded with $b > 0$ (see Fig. 1). Here $\boldsymbol{\nu}$ represents the outward unit normal vector on the boundary of Ω_∞, and Θ is a simply connected and smooth (or piecewise smooth with no reentrance corners) bounded domain in $\mathbb{R}^2$. We assume that the cross-section denoted by Θ of the semi-infinite

E-mail address: sikim@khu.ac.kr.

http://dx.doi.org/10.1016/j.jmaa.2017.03.082

Please cite this article in press as: S. Kim, Analysis of the non-reflecting boundary condition for the time-harmonic electromagnetic wave propagation in waveguides, J. Math. Anal. Appl. (2017), http://dx.doi.org/10.1016/j.jmaa.2017.03.082

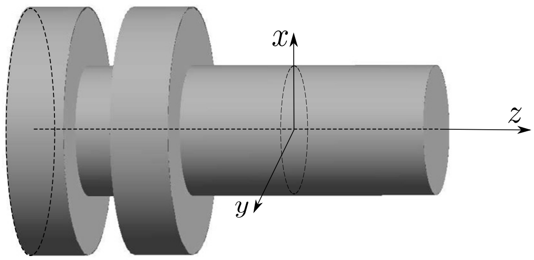

Fig. 1. Semi-infinite waveguide Ω_∞.

waveguide Ω_∞ perpendicular to the z-axis is uniform for $z > -b$. Electric fields for this model problem are confined within the perfectly conducting boundary $\partial\Omega_\infty$ and propagate along the z-axis. Also, ω stands for a positive angular frequency and $\epsilon \in L^\infty(\Omega_\infty)$ is the electric permittivity satisfying $\Im(\epsilon) \geq 0$ and $\Re(\epsilon) > \epsilon_0$ for a positive constant $\epsilon_0 > 0$ and $\epsilon - 1$ has a compact support, saying $\epsilon = 1$ for $z > -b$. In addition, $\boldsymbol{J}$ represents the current source density satisfying $\nabla \cdot \boldsymbol{J} = 0$ and vanishing for $z > -b$.

The numerical study for electromagnetic wave propagation in the unbounded domain Ω_∞ requires domain truncation to a finite computational domain e.g., $\Omega = \Omega_\infty \cap \{z < 0\}$ with a non-reflecting boundary condition on the fictitious boundary $\Gamma_E = \Theta \times \{0\}$, which can guarantee that the reduced problem has the same radiation solution without reflection. The goal of the paper is to study the non-reflecting radiation boundary condition on Γ_E satisfied by radiating electric fields in the semi-infinite waveguide and analyze the well-posedness of the truncated problem supplemented with the non-reflecting radiation boundary condition. This result will play a crucial role for developing absorbing boundary conditions such as perfectly matched layers (PMLs) and complete radiation boundary conditions (CRBCs) and studying their convergence to the exact radiation condition in a subsequent work.

In case of no inhomogeneity inclusion in waveguides, the Maxwell's equations can be reduced to the scalar Helmholtz equation. It coincides with the acoustic wave propagation problem, for which the Dirichlet-to-Neumann operator can give an approach for defining the non-reflecting boundary condition. This simplified model has been extensively investigated as well as ones with approximate absorbing boundary conditions including truncated DtN [5,18,26] PML [4] and CRBCs [21–23]. When ϵ is not constant, however, components of electromagnetic fields can not be separated in general and the full-wave analysis based on the vector equations such as (1.1) is required. To the best of the author's knowledge, radiation conditions for vector electromagnetic fields in waveguides have not been studied, although non-reflecting radiation boundary conditions for exterior scattering problems have been studied in [2] with the Dirichlet-to-Neumann operator and in [24,27] based on the electric-to-magnetic operator.

On the other hand, it is worth noting that there are studies about electromagnetic wave propagation in waveguides with more general ϵ, that can be a function of x and y but does not have a variation along the axis of waveguides. In such a case, propagation constants for a given wavenumber are involved in a non-selfadjoint eigenvalue problem, which is a more difficult problem than the homogeneous one. Thus, the research for the general model is focused on computation of propagation constants of the Maxwell's equations for a given wavenumber as done in [13,28,29]. Some studies [6,20] are interested in possible wavenumbers for a given propagation constant. Also, a study on electromagnetic wave propagation in periodic media can be found in [1].

This paper consists of two parts. One is devoted to defining the electric-to-magnetic operator suitable for electromagnetic fields in waveguides by using a series representation of radiating electromagnetic fields. For this, it is required to understand the tangential traces and tangential component traces of functions belonging to $\boldsymbol{H}(curl, \Omega)$ studied in [3,11,10]. In particular, characterization of $\boldsymbol{\nu} \times \boldsymbol{u}$ and $\boldsymbol{u}$ on Γ_E, when $\boldsymbol{\nu} \times \boldsymbol{u}$ for $\boldsymbol{u} \in \boldsymbol{H}(curl, \Omega)$ vanishes on $\partial\Theta \times (-b, 0)$, will be presented (see e.g., [8,9]). The other is for establishing the well-posedness of the truncated problem supplemented with the non-reflecting radiation boundary condition

Please cite this article in press as: S. Kim, Analysis of the non-reflecting boundary condition for the time-harmonic electromagnetic wave propagation in waveguides, J. Math. Anal. Appl. (2017), http://dx.doi.org/10.1016/j.jmaa.2017.03.082