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Hilbert matrix on spaces of Bergman-type

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ABSTRACT

It is well known (see [8,14]) that the Libera operator $\mathcal{L}$ is bounded on the Besov space $H^{p,q,\alpha}_\nu$ if and only if $0 < \kappa_{p,\alpha,\nu} := \nu - \alpha - \frac{1}{p} + 1$. We prove unexpected results: the Hilbert matrix operator H, as well as the modified Hilbert operator $\widetilde{H}$, is bounded on $H^{p,q,\alpha}_\nu$ if and only if $0 < \kappa_{p,\alpha,\nu} < 1$. In particular, H, as well as $\widetilde{H}$, is bounded on the Bergman space $A^{p,\alpha}$ if and only if $1 < \alpha + 2 < p$ and is bounded on the Dirichlet space $\mathcal{D}^p_\alpha = A^{p,\alpha}_1$ if and only if $\max\{-1, p-2\} < \alpha < 2p-2$. Our results are substantial improvement of [11, Theorem 3.1] and of [6, Theorem 5].

1. Introduction

Let $\mathcal{H}(\mathbb{D})$ be the space of all functions holomorphic in the unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ endowed with the topology of uniform convergence on compact subsets of $\mathbb{D}$.

For $0 < p \le \infty$, Hardy space H^p is the space of all functions $f \in \mathcal{H}(\mathbb{D})$ for which

$$\|f\|_{H^p} = \|f\|_p = \sup_{0\le r<1} M_p(r, f) < \infty,$$

where

$$M_p(r, f) = \left(\frac{1}{2\pi}\int_0^{2\pi} |f(re^{it})|^p dt\right)^{\frac{1}{p}}, \ 0 < p < \infty;$$

$$M_\infty(r, f) = \sup_{0\le t<2\pi} |f(re^{it})|.$$

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A function $f \in \mathcal{H}(\mathbb{D})$ is said to belong to the mixed norm space $H^{p,q,\alpha}$, $0 < p, q \le \infty$, $0 < \alpha < \infty$, if

$$\|f\|_{H^{p,q,\alpha}} = \|f\|_{p,q,\alpha} = \left(\int_0^1 M_p^q(r,f)(1-r)^{q\alpha-1} dr \right)^{\frac{1}{q}} < \infty, \; 0 < q < \infty;$$

$$\|f\|_{H^{p,\infty,\alpha}} = \|f\|_{p,\infty,\alpha} = \sup_{0 \le r < 1} (1-r)^{\alpha} M_p(r,f) < \infty.$$

The normalized Lebesgue area measure on $\mathbb{D}$ will be denoted by A, i.e.,

$$dA(z) = \frac{1}{\pi} dxdy = \frac{1}{\pi} r dr d\theta, \; z = x + iy = re^{i\theta}.$$

Recall that for $0 < p < \infty$ and $\alpha > -1$, the (weighted) Bergman space $A^{p,\alpha} = A^{p,\alpha}(\mathbb{D})$ is the space of analytic functions in $L^p(\mathbb{D}, dA_\alpha)$ where

$$dA_\alpha(z) = (\alpha+1)\left(1-|z|^2\right)^{\alpha} dA(z).$$

If $f \in L^p(\mathbb{D}, dA_\alpha) \cap \mathcal{H}(\mathbb{D})$, we write

$$\|f\|_{A^{p,\alpha}} = \|f\|_{p,\alpha} = \left(\int_{\mathbb{D}} |f(z)|^p dA_\alpha(z) \right)^{\frac{1}{p}}.$$

It is easy to check that $f \in A^{p,\alpha}$ if and only if

$$\|f\|^p_{p,p,\frac{\alpha+1}{p}} = \int_0^1 (1-r)^{\alpha} M_p^p(r,f) dr < \infty.$$

Note also that $\|f\|_{p,\alpha}$ is comparable to $\|f\|_{p,p,\frac{\alpha+1}{p}}$. Hence $A^{p,\alpha} = H^{p,p,\frac{\alpha+1}{p}}$. Simply $A^p = A^{p,0}$ are (unweighted) Bergman spaces.

For $t \in \mathbb{R}$ we write D^t for the sequence $\{(n+1)^t\}$, for all $n \ge 0$. If $\lambda = \{\lambda_n\}_{n=0}^{\infty}$ is a sequence and X is a sequence space (by identifying the holomorphic function $f(z) = \sum_{n=0}^{\infty} \widehat{f}(n) z^n$ with the sequence $\{\widehat{f}(n)\}_{n=0}^{\infty}$ we may consider the spaces of holomorphic functions as sequence spaces), we write

$$\lambda X = \left\{ \lambda * x = \{\lambda_n x_n\}_{n=0}^{\infty} : x = \{x_n\}_{n=0}^{\infty} \in X \right\}.$$

For example $\{a_n\}_{n=0}^{\infty} \in D^1 l^1$ if and only if $\sum_{n=0}^{\infty} \frac{|a_n|}{n+1} < \infty$. The space $D^t H^{p,q,\alpha}$, for $t \ne 0$, will also be denoted by $H^{p,q,\alpha}_{-t}$.

Among the spaces $H^{p,q,\alpha}_s$, $0 < s < \infty$, the spaces $H^{p,q,1}_{1+s}$ are of independent interest, and are known as Besov spaces for $0 < q < \infty$, and as Lipschitz spaces when $q = \infty$.

We note that in [13] the spaces of functions $f \in \mathcal{H}(\mathbb{D})$ such that $D^n f \in H^{p,q,n-\alpha}$, $\alpha \in \mathbb{R}$ (equivalently $f^{(n)} \in H^{p,q,n-\alpha}$) for some (any) nonnegative integer n such that $n - \alpha > 0$ are called Besov spaces and they are denoted by $\mathcal{B}^{p,q}_\alpha$. Comparing with the definitions given above, $\mathcal{B}^{p,q}_\alpha = H^{p,q,-\alpha}$, for $\alpha < 0$, and $\mathcal{B}^{p,q}_\alpha = H^{p,q,1}_{1+\alpha}$, for $\alpha > 0$.

The Hilbert matrix is an infinite matrix H whose entries are $a_{n,k} = \frac{1}{n+k+1}$. We note that H as an operator on ℓ^2 was first studied by Magnus [12]. It can be also viewed as an operator on spaces of holomorphic