J. Math. Anal. Appl. 453 (2017) 410–433

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Existence for the compressible magnetohydrodynamic equations with vacuum

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ARTICLE INFO

Article history:
Received 13 February 2017
Available online 6 April 2017
Submitted by D. Wang

Keywords:
MHD
Classical solutions
Vacuum

ABSTRACT

In this paper, the 3-D compressible MHD equations without thermal conductivity are considered. The existence of unique local classical solutions to the initial-boundary value problem with Dirichlet or Navier-Slip boundary condition is established when the initial data are arbitrarily large, contains vacuum and satisfies some initial layer compatibility condition. The initial density needs not to be bounded away from zero and may vanish in some open set.

1. Introduction

Magnetohydrodynamics is that part of the mechanics of continuous media which studies the motion of electrically conducting media in the presence of a magnetic field. The dynamic motion of fluid and magnetic field interact strongly on each other, so the hydrodynamic and electrodynamic effects are coupled. The applications of magnetohydrodynamics cover a wide range of physical objects, from liquid metals to cosmic plasmas, for example, the intensely heated and ionized fluids in an electromagnetic field in astrophysics and plasma physics. In 3-D space, the compressible magnetohydrodynamic (MHD) equations in a domain $\Omega \subset \mathbb{R}^3$ can be written as

$$\begin{cases} H_t - \mathrm{rot}(u \times H) = -\mathrm{rot}\Big(\dfrac{1}{\sigma}\mathrm{rot}H\Big), \\ \mathrm{div}H = 0, \\ \rho_t + \mathrm{div}(\rho u) = 0, \\ (\rho u)_t + \mathrm{div}(\rho u \otimes u) + \nabla P = \mathrm{div}\mathbb{T} + \mathrm{rot}H \times H, \\ (\rho e)_t + \mathrm{div}(\rho e u) - \kappa \triangle \theta + P\mathrm{div}u = \mathrm{div}(u\mathbb{T}) - u\mathrm{div}\mathbb{T} + \dfrac{1}{\sigma}|\mathrm{rot}H|^2. \end{cases} \tag{1.1}$$

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http://dx.doi.org/10.1016/j.jmaa.2017.04.007
0022-247X/

In this system, $x \in \Omega$ is the spatial coordinate; $t \geq 0$ is the time variable; $H = (H^{(1)}, H^{(2)}, H^{(3)}) \in \mathbb{R}^3$ is the magnetic field; ρ is the density; $u = (u^{(1)}, u^{(2)}, u^{(3)}) \in \mathbb{R}^3$ is the velocity of fluids; e is the specific internal energy; $\text{rot}H = \nabla \times H$ denotes the rotation of the magnetic field; $0 < \sigma \leq \infty$ is the electric conductivity coefficient; $\kappa \geq 0$ is the thermal conductivity coefficient; P is the pressure satisfying

$$P = R\rho\theta = (\gamma - 1)\rho e, \text{ with } \gamma > 1, \tag{1.2}$$

where θ is the absolute temperature, R is a positive constant, γ is the adiabatic index; $\mathbb{T}$ is the stress tensor given by

$$\mathbb{T} = 2\mu D(u) + \lambda \text{div} u \mathbb{I}_3, \quad D(u) = \frac{\nabla u + (\nabla u)^\top}{2}, \tag{1.3}$$

where $D(u)$ is the deformation tensor, $\mathbb{I}_3$ is the 3×3 unit matrix, $\mu > 0$ is the shear viscosity coefficient, $\lambda + \frac{2}{3}\mu \geq 0$ is the bulk viscosity coefficient. Although the electric field E doesn't appear in system (1.1), it is indeed induced according to a relation

$$E = \frac{1}{\sigma}\text{rot}H - u \times H$$

by moving the conductive flow in the magnetic field.

However, when $\kappa = 0$, system (1.4) for classical solutions can be written into

$$\begin{cases} H_t - \text{rot}(u \times H) = -\text{rot}\Big(\dfrac{1}{\sigma}\text{rot}H\Big), \\ \text{div}H = 0, \\ \rho_t + \text{div}(\rho u) = 0, \\ (\rho u)_t + \text{div}(\rho u \otimes u) + \nabla P = \text{div}\mathbb{T} + \text{rot}H \times H, \\ P_t + u \cdot \nabla P + \gamma P \text{div} u = (\gamma - 1)(\text{div}(u\mathbb{T}) - u\text{div}\mathbb{T}) + \dfrac{\gamma - 1}{\sigma}|\text{rot}H|^2. \end{cases} \tag{1.4}$$

The initial condition is given as

$$(H, \rho, u, P)|_{t=0} = (H_0(x), \rho_0(x), u_0(x), P_0(x)), \ x \in \Omega. \tag{1.5}$$

We only consider the following two types of boundary conditions in this paper for simplicity:

(1) Dirichlet boundary condition for (u, H): $\Omega \in \mathbb{R}^3$ is a bounded smooth domain and

$$u|_{\partial\Omega} = 0, \quad \text{when} \quad \sigma = +\infty; \quad (u, H)|_{\partial\Omega} = (0, 0), \quad \text{when} \ \ 0 < \sigma < +\infty. \tag{1.6}$$

(2) Navier-slip boundary condition for (u, H): $\Omega \in \mathbb{R}^3$ is bounded, simply connected, smooth domain, and

$$(u \cdot n, \text{rot}u \times n, H \cdot n, \text{rot}H \times n)|_{\partial\Omega} = (0, 0, 0, 0), \quad \text{when} \ \ 0 < \sigma < +\infty, \tag{1.7}$$

where $\nabla \times u$ denotes the vorticity field of fluids and n is the unit outer normal vector of $\partial\Omega$. Actually, the similar existence result when $\Omega = \mathbb{R}^3$, half space $\mathbb{R}^2 \times \mathbb{R}^+$ or exterior domain with smooth boundary can be obtained via the similar argument used in this paper.

Throughout this paper, we adopt the following simplified notations for the standard homogeneous and inhomogeneous Sobolev space: