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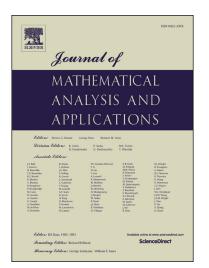
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Sandwich theorem for *m*-convex functions

Janusz Matkowski and Małgorzata Wróbel

ABSTRACT. A sandwich theorem for *m*-convex functions with $m \in [0, 1]$ is proved. The counterexamples to a sandwich type result for *m*-convex functions with 0 < m < 1, presented in [3], are given.

1. Introduction

The following sandwich type result has been proved in [1]: the real functions f and g defined on interval I (or a convex set in \mathbb{R}^k) satisfy the inequality

$$f(tx + (1 - t)y) \le tg(x) + (1 - t)g(y)$$

for all $x, y \in I$ and $t \in [0, 1]$ if, and only if, there is a real convex function h defined on I such that

$$f(x) \le h(x) \le g(x)$$

for all $x \in I$.

In a recent paper [3], (Theorem 8), the authors formulate and prove the counterpart for *m*-convex functions (the notion introduced by Toader [5]) which reads as follows. Let $0 \le m < 1$. The real functions f and g defined on $(0, \infty)$ satisfy the inequality

$$f(tx + m(1 - t)y) \le tg(x) + m(1 - t)g(y)$$

for all $x, y \in (0, \infty)$ and $t \in [0, 1]$ if, and only if, there is a real m-convex function h defined on $(0, \infty)$, i.e., satisfying the inequality

$$h(tx + m(1 - t)y) \le th(x) + m(1 - t)h(y)$$

for all $x, y \in (0, \infty)$ and $t \in [0, 1]$ such that

$$f(x) \le h(x) \le g(x)$$

for all $x \in I$.

In view of Theorem 1 in [1], this result holds true for m = 1. In section 2, using a simple argument, we prove that it remains true if m = 0. In section 3 we give a counterexample showing that this result is false for every $m \in (0, 1)$.

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