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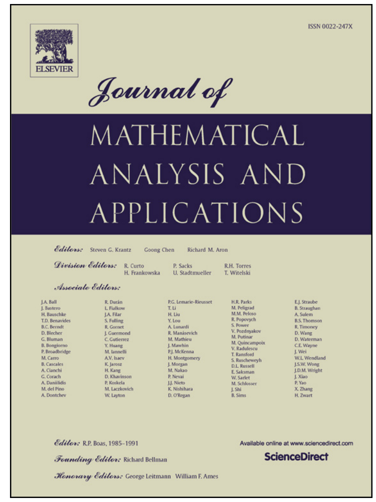

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Sandwich theorem for m-convex functions

Janusz Matkowski and Małgorzata Wróbel

Abstract. A sandwich theorem for m-convex functions with $m \in [0,1]$ is proved. The counterexamples to a sandwich type result for m-convex functions with $0 < m < 1$, presented in [**3**], are given.

1. Introduction

The following sandwich type result has been proved in [**1**]: *the real functions f and g defined on interval I (or a convex set in R^k) satisfy the inequality*

$$f(tx + (1-t)y) \leq tg(x) + (1-t)g(y)$$

for all $x, y \in I$ and $t \in [0,1]$ if, and only if, there is a real convex function h defined on I such that

$$f(x) \leq h(x) \leq g(x)$$

for all $x \in I$.

In a recent paper [**3**], (Theorem 8), the authors formulate and prove the counterpart for m-convex functions (the notion introduced by Toader [**5**]) which reads as follows. *Let $0 \leq m < 1$. The real functions f and g defined on $(0,\infty)$ satisfy the inequality*

$$f(tx + m(1-t)y) \leq tg(x) + m(1-t)g(y)$$

for all $x, y \in (0,\infty)$ and $t \in [0,1]$ if, and only if, there is a real m-convex function h defined on $(0,\infty)$, i.e., satisfying the inequality

$$h(tx + m(1-t)y) \leq th(x) + m(1-t)h(y)$$

for all $x, y \in (0,\infty)$ and $t \in [0,1]$ such that

$$f(x) \leq h(x) \leq g(x)$$

for all $x \in I$.

In view of Theorem 1 in [**1**], this result holds true for $m = 1$. In section 2, using a simple argument, we prove that it remains true if $m = 0$. In section 3 we give a counterexample showing that this result is false for every $m \in (0,1)$.

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