

On anisotropic singularities for semi-linear elliptic equations in $\mathbb{R}^2$

Ying Wang, Huyuan Chen

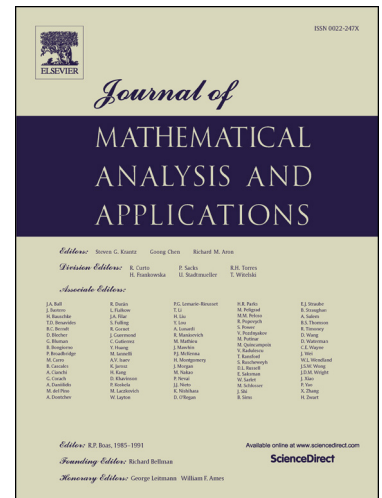
PII: S0022-247X(17)30195-6
 DOI: <http://dx.doi.org/10.1016/j.jmaa.2017.02.045>
 Reference: YJMAA 21170

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 25 September 2015

Please cite this article in press as: Y. Wang, H. Chen, On anisotropic singularities for semi-linear elliptic equations in $\mathbb{R}^2$, *J. Math. Anal. Appl.* (2017), <http://dx.doi.org/10.1016/j.jmaa.2017.02.045>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



ON ANISOTROPIC SINGULARITIES FOR SEMI-LINEAR ELLIPTIC EQUATIONS IN $\mathbb{R}^2$

YING WANG¹ AND HUYUAN CHEN²

Department of Mathematics, Jiangxi Normal University,
Nanchang, Jiangxi 330022, PR China

Abstract. In this note, we study weak solutions of

$$\begin{aligned} -\Delta u + g(u) &= 8\pi \frac{\partial \delta_0}{\partial x_2} \quad \text{in } \mathbb{R}^2, \\ u(x) &\rightarrow 0 \quad \text{as } |x| \rightarrow +\infty, \end{aligned} \tag{1}$$

where $g(s) = 2\frac{e^s-1}{e^s+1}$ is an odd, increasing and bounded function and $\frac{\partial \delta_0}{\partial x_2}$ is defined in the sense of distribution

$$\left\langle \frac{\partial \delta_0}{\partial x_2}, \xi \right\rangle = \frac{\partial \xi(0)}{\partial x_2} \quad \text{for } \xi \in C_c^1(\mathbb{R}^2).$$

We obtain that problem (1) admits a unique, x_2 -odd weak solution, which has anisotropic singularity $4\frac{x_2}{|x|^2}$ near the origin.

1. INTRODUCTION

Vortices appear in various planar condensed-matter systems and have important applications in many fundamental areas of physics including superconductivity [1, 10, 13], particle physics [12], optics [5] and cosmology [21]. The study of multiple charges vortex construction in gauged field theory was initiated by Taubes [13, 17, 18] regarding the existence and uniqueness of static solutions of the Abelian Higgs model. Later on, Schroers [16] extended the classical $O(3)$ sigma model solved by Belavin-Polyakov [2] to incorporate an Abelian gauged field and allow the existence of vortices of opposite local charges so that the vortices of negative local charges viewed as poles of a complex scalar field u makes contribute to, but those of positive local charges viewed as zero of u do not affect, the total energy, although they give some magnetic manifestation for their existence [22]. In fact, these peculiar properties are all due to the absence of symmetry breaking and in order to obtain vortices of opposite magnetic alignments with an energy that takes account of both type of vortices, it suffices to impose a broken symmetry. After that, Yang in [23] established an Abelian field theory model that allows the coexistence of vortices and anti-vortices, showed how vortices and anti-vortices with the coupling of gravity, namely, cosmic strings and anti-strings, can be constructed in the Abelian gauged field model. More precisely, the coexistence model in [23] states as follows. Let $q_1, q_2, \dots, q_M$ and $p_1, p_2, \dots, p_N$ be the locations in $\mathbb{R}^2$ of vortices and anti-vortices of unit charge respectively, from Bogomol'nyi equations

$$D_1 v + iD_2 v = 0,$$

$$F_{12} = e^\eta \frac{1-|v|^2}{1+|v|^2},$$

¹yingwang00@126.com

²chenhuyuan@yeah.net

AMS Subject Classifications: 35R06, 35B40, 35Q60.

Key words: Vortex and Anti-Vortex; Existence and Uniqueness; Anisotropic singularity.

Download English Version:

<https://daneshyari.com/en/article/5775001>

Download Persian Version:

<https://daneshyari.com/article/5775001>

[Daneshyari.com](https://daneshyari.com)