

Accepted Manuscript

Some evaluation of parametric Euler sums

Ce Xu

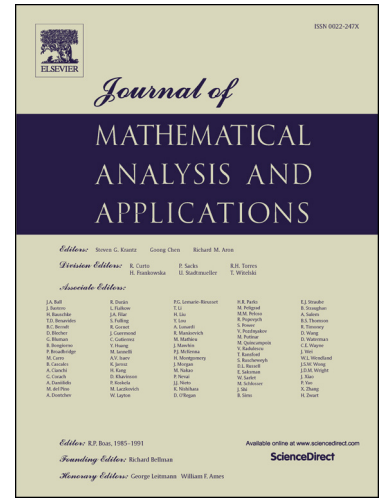
PII: S0022-247X(17)30197-X
DOI: <http://dx.doi.org/10.1016/j.jmaa.2017.02.047>
Reference: YJMAA 21172

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 24 June 2015

Please cite this article in press as: C. Xu, Some evaluation of parametric Euler sums, *J. Math. Anal. Appl.* (2017), <http://dx.doi.org/10.1016/j.jmaa.2017.02.047>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



Some evaluation of parametric Euler sums

Ce Xu*

School of Mathematical Sciences, Xiamen University
Xiamen 361005, P.R. China

Abstract In this paper, by using the method of Contour Integral Representations and the Theorem of Residues and integral representations of series, we discuss the analytic representations of parametric Euler sums that involve harmonic numbers through zeta values and rational function series, either linearly or nonlinearly. Furthermore, we give explicit formulae for several parametric quadratic and cubic sums in terms of zeta values and rational series. Moreover, some interesting new consequences and illustrative examples are considered.

Keywords Harmonic number; Euler sum; Riemann zeta function; Hurwitz zeta function.

AMS Subject Classifications (2010): 11M06; 11M32.

1 Introduction

Throughout this article we will use the following definitions and notations. Let $\mathbb{N} := \{1, 2, 3, \dots\}$ be the set of natural numbers, and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$, $\mathbb{N} \setminus \{1\} := \{2, 3, 4, \dots\}$. In this paper, harmonic numbers, alternating harmonic numbers and their generalizations are classically defined by

$$\zeta_n(k) := \sum_{j=1}^n \frac{1}{j^k}, \quad L_n(k) := \sum_{j=1}^n \frac{(-1)^{j-1}}{j^k}, \quad k \in \mathbb{N}, \quad (1.1)$$

where $H_n := \zeta_n(1) = \sum_{j=1}^n \frac{1}{j}$ is classical harmonic number and the empty sum $\zeta_0(m)$ is conventionally understood to be zero. The subject of this paper is Euler sums, which are the infinite sums whose general term is a product of harmonic numbers and alternating harmonic numbers of index n and a power of n^{-1} or $((-1)^{n-1}n^{-1})$. Hence, more generally we can define the Euler sums by the series ([18,19])

$$\sum_{n=1}^{\infty} \frac{\prod_{i=1}^{m_1} \zeta_n^{q_i}(k_i) \prod_{j=1}^{m_2} L_n^{l_j}(h_j)}{n^p}, \quad \sum_{n=1}^{\infty} \frac{\prod_{i=1}^{m_1} \zeta_n^{q_i}(k_i) \prod_{j=1}^{m_2} L_n^{l_j}(h_j)}{n^p} (-1)^{n-1}, \quad (1.2)$$

where $m_1, m_2, q_i, k_i, h_j, l_j, p (p \geq 2)$ are positive integers. The quantity $w := \sum_{i=1}^{m_1} (k_i q_i) + \sum_{j=1}^{m_2} (h_j l_j) +$

p is called the weight, the quantity $k := \sum_{i=1}^{m_1} q_i + \sum_{j=1}^{m_2} l_j$ is called the degree.

*Corresponding author. Email: xuce1242063253@163.com

Download English Version:

<https://daneshyari.com/en/article/5775002>

Download Persian Version:

<https://daneshyari.com/article/5775002>

[Daneshyari.com](https://daneshyari.com)