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General decay and blow up of solutions for a system of viscoelastic wave equations with nonlinear boundary source terms

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A R T I C L E   I N F O

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In this work, an initial boundary value problem for a system of viscoelastic wave equations with nonlinear boundary source term of the form

$$(u_i)_{tt} - \Delta u_i - \Delta(u_i)_{tt} + \int_0^t g_i(t-s)\Delta u_i(s)ds - \Delta(u_i)_t = 0, \quad \text{in} \quad \Omega \times (0,T),$$

$$u_i(x,0) = \varphi_i(x), \quad (u_i)_t(x,0) = \psi_i(x), \quad \text{in} \quad \Omega,$$

$$u_i(x,t) = 0, \quad \text{on} \quad \Gamma_0 \times (0,T),$$

$$\partial_\nu(u_i)_{tt} + \partial_\nu u_i - \int_0^t g_i(t-s)\partial_\nu u_i(s)ds + \partial_\nu(u_i)_t + f_i(u) = 0, \quad \text{on} \quad \Gamma_1 \times (0,T),$$

where $i = 1, ..., l$ ($l \geq 2$) is considered in a bounded domain Ω in $\mathbb{R}^N$ ($N \geq 1$). By the Faedo–Galerkin approximation method we obtain existence and uniqueness of weak solutions. Under appropriate assumptions on initial data and the relaxation functions, we establish general decay and blow up results associated to solution energy. Estimates for lifespan of solutions are also given.

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1. Introduction

In this paper, we investigate general stability and instability behavior of solutions for the system of nonlinear wave equations

$$(u_i)_{tt} - \Delta u_i - \Delta(u_i)_{tt} + \int_0^t g_i(t-s)\Delta u_i(s)ds - \Delta(u_i)_t = 0, \quad \text{in} \quad \Omega \times (0,T), \tag{1.1}$$

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supplemented by the initial conditions

$$u_i(x,0) = \varphi_i(x), \quad (u_i)_t(x,0) = \psi_i(x), \quad \text{in} \quad \Omega, \tag{1.2}$$

and the following boundary conditions:

$$u_i(x,t) = 0, \quad \text{on} \quad \Gamma_0 \times (0,T), \tag{1.3}$$

$$\partial_\nu (u_i)_{tt} + \partial_\nu u_i - \int_0^t g_i(t-s)\partial_\nu u_i(s)ds + \partial_\nu (u_i)_t + f_i(u) = 0, \quad \text{on} \quad \Gamma_1 \times (0,T), \tag{1.4}$$

where $i = 1, \dots, l$ $(l \geq 2)$, $u = (u_1, \dots, u_l)$, Ω is a bounded domain in $\mathbb{R}^N (N \geq 1)$ with a smooth boundary $\partial\Omega = \Gamma_0 \cup \Gamma_1, \overline{\Gamma}_0 \cap \overline{\Gamma}_1 = \emptyset$, Γ_0 and Γ_1 are closed with positive measures, ν is the unit outward normal to $\partial\Omega$, $T > 0$ is a constant, g_i and f_i are nonlinear functions.

In recent decades, viscoelastic wave equations have attached attention of many authors. Dafermos in [9] considered an abstract Volterra equation and proved an existence-uniqueness result and investigated the behavior of solutions at infinity and the results were then applied to viscoelasticity. Later, the author investigated asymptotic stability of solutions of a one dimensional viscoelastic wave equation in [10] under more stringent assumptions on the function spaces in comparison with the ones in [9]. Such results then were the origin of significant works in connecting with the long time behavior studies in viscoelasticity. In this regard, we may refer to the works by Hrusa ([16], 1985), Rivera ([26], 1994) and more related studies in [7,11,27,28].

Cavalcanti et al. [3] considered the equation

$$|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g(t-\tau)\Delta u(\tau)d\tau - \gamma\Delta u_t = 0, \qquad x \in \Omega, \quad t > 0, \tag{1.5}$$

subject to Dirichlet boundary conditions. Taking $0 \leq \rho \leq \frac{2}{n-2}$ if $n \geq 3$ or $\rho > 0$ if $n = 1, 2$ and assuming that the kernel g decays exponentially, the authors obtained global existence of solutions in the case $\gamma \geq 0$. They also proved that the solution energy decays exponentially when $\gamma > 0$. Messaoudi and Tatar [25] later extended this result to the case $\gamma = 0$. In [24], by introducing a new functional and a potential well method, they obtained the global existence of solutions and uniform decay of energy for

$$|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + f(u) = 0, \qquad x \in \Omega, \quad t > 0, \tag{1.6}$$

with $f(u) = -u|u|^{p-2}$ and Dirichlet boundary conditions if the initial data are in some stable set. Wu [30] extended this result in presence of nonlinear damping terms under appropriate assumptions on the relaxation function and the initial data. He then also proved an arbitrary decay result in [31] for (1.6) with $f(u) = u|u|^p$ for a class of kernel function g without setting the function g itself to be of exponential (polynomial) type, which is a necessary condition for the exponential (polynomial) decay of the solution energy for the viscoelastic problem. In the case $\rho = 0$ and in the absence of dispersions, Cavalcanti et al. in [5] investigated

$$u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + a(x)u_t + |u|^\gamma u = 0, \qquad x \in \Omega, \quad t > 0,$$

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