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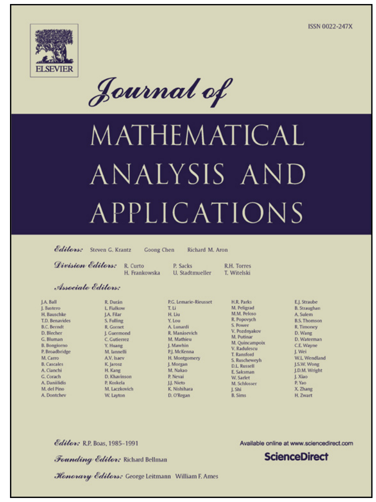


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# INVARIANCE OF CLOSED CONVEX CONES FOR STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT. The goal of this paper is to clarify when a closed convex cone is invariant for a stochastic partial differential equation (SPDE) driven by a Wiener process and a Poisson random measure, and to provide conditions on the parameters of the SPDE, which are necessary and sufficient.

## 1. INTRODUCTION

Consider a semilinear stochastic partial differential equation (SPDE) of the form

$$\begin{cases} dr_t &= (Ar_t + \alpha(r_t))dt + \sigma(r_t)dW_t + \int_E \gamma(r_{t-}, x)(\mu(dt, dx) - F(dx)dt) \\ r_0 &= h_0 \end{cases} \tag{1.1}$$

driven by a trace class Wiener process $W$ and a Poisson random measure $\mu$ on some mark space $E$ with compensator $dt \otimes F(dx)$. The state space of the SPDE (1.1) is a separable Hilbert space $H$, and the operator $A$ is the generator of a strongly continuous semigroup $(S_t)_{t \geq 0}$ on $H$. We refer to Section 2 for more details concerning the mathematical framework.

In applications, one is often interested in the question when a certain subset of the state space is invariant for the SPDE (1.1), and frequently it turns out that this subset is a closed convex cone. For example, when modeling the evolution of interest rate curves, a desirable feature is that the model produces nonnegative interest curves; or when modeling multiple yield curves, it is desirable to have spreads which are ordered with respect to different tenors.

In order to translate these ideas into mathematical terms, let $K \subset H$ be a closed convex cone of the state space $H$. We say that the cone $K$ is invariant for the SPDE (1.1) if for each starting point $h_0 \in K$ the solution process $r$ to (1.1) stays in $K$. The goal of this paper is to clarify when the cone $K$ is invariant for the SPDE (1.1), and to provide conditions on the parameters $(A, \alpha, \sigma, \gamma)$ – or, equivalently, on $((S_t)_{t \geq 0}, \alpha, \sigma, \gamma)$ – of the SPDE (1.1), which are necessary and sufficient.

Stochastic invariance of a given subset $K \subset H$ for jump-diffusion SPDEs (1.1) has already been studied in the literature, mostly for diffusion SPDEs

$$\begin{cases} dr_t &= (Ar_t + \alpha(r_t))dt + \sigma(r_t)dW_t \\ r_0 &= h_0 \end{cases} \tag{1.2}$$

without jumps. The classes of subsets $K \subset H$, for which stochastic invariance has been investigated, can roughly be divided as follows:

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