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Invariance of closed convex cones for stochastic partial differential equations

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### ACCEPTED MANUSCRIPT

#### INVARIANCE OF CLOSED CONVEX CONES FOR STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

#### STEFAN TAPPE

ABSTRACT. The goal of this paper is to clarify when a closed convex cone is invariant for a stochastic partial differential equation (SPDE) driven by a Wiener process and a Poisson random measure, and to provide conditions on the parameters of the SPDE, which are necessary and sufficient.

#### 1. INTRODUCTION

Consider a semilinear stochastic partial differential equation (SPDE) of the form

(1.1)  $\begin{cases}
 dr_t = (Ar_t + \alpha(r_t))dt + \sigma(r_t)dW_t + \int_E \gamma(r_{t-}, x)(\mu(dt, dx) - F(dx)dt) \\
 r_0 = h_0
\end{cases}$ 

driven by a trace class Wiener process W and a Poisson random measure  $\mu$  on some mark space E with compensator  $dt \otimes F(dx)$ . The state space of the SPDE (1.1) is a separable Hilbert space H, and the operator A is the generator of a strongly continuous semigroup  $(S_t)_{t\geq 0}$  on H. We refer to Section 2 for more details concerning the mathematical framework.

In applications, one is often interested in the question when a certain subset of the state space is invariant for the SPDE (1.1), and frequently it turns out that this subset is a closed convex cone. For example, when modeling the evolution of interest rate curves, a desirable feature is that the model produces nonnegative interest curves; or when modeling multiple yield curves, it is desirable to have spreads which are ordered with respect to different tenors.

In order to translate these ideas into mathematical terms, let  $K \subset H$  be a closed convex cone of the state space H. We say that the cone K is invariant for the SPDE (1.1) if for each starting point  $h_0 \in K$  the solution process r to (1.1) stays in K. The goal of this paper is to clarify when the cone K is invariant for the SPDE (1.1), and to provide conditions on the parameters  $(A, \alpha, \sigma, \gamma)$  – or, equivalently, on  $((S_t)_{t>0}, \alpha, \sigma, \gamma)$  – of the SPDE (1.1), which are necessary and sufficient.

Stochastic invariance of a given subset  $K \subset H$  for jump-diffusion SPDEs (1.1) has already been studied in the literature, mostly for diffusion SPDEs

(1.2) 
$$\begin{cases} dr_t = (Ar_t + \alpha(r_t))dt + \sigma(r_t)dW_t \\ r_0 = h_0 \end{cases}$$

without jumps. The classes of subsets  $K \subset H$ , for which stochastic invariance has been investigated, can roughly be divided as follows:

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Key words and phrases. Stochastic partial differential equation, closed convex cone, stochastic invariance, parallel function.

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