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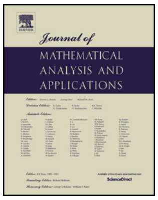

Bistability in a model of grassland and forest transition

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ABSTRACT

It is acknowledged that the role of fire and climate in determining savanna and forest distributions requires comprehensive theoretical reevaluation. In this paper, an ordinary differential equation model describing the grassland and forest transition is considered. We use a modified Hill function as the nonlinear response of fire frequency to grass abundance, and the global dynamics of the model is completely classified. It is shown that any solution converges to an equilibrium, and the system has one, two or three coexistence equilibria depending on parameter values. Our results provide precise parameter range of bistability.

1. Introduction

Vegetation growth responding to climate changes is one of the core topics regarding global environment change. There is growing evidence within the environmental science community that our planet has entered a potentially catastrophic phase of human-induced global climate warming [9,10]. Human activity has a huge impact on the changing of vegetation. Excessive deforestation, tourism development, urban expansion all influence the amount of tree cover and destroy huge area of forest.

The tree cover is influenced by complex interaction between climate, human activity, animals, soil and many other unexpected disturbances. It is interesting that in some area where the environmental conditions suggest that forest should dominant, it turns out to be savanna prevailing [19]. Also an intermediate level of tree cover rarely happens, and there is a transition from the savanna state to the forest state when the environmental condition changes. This suggests that in the ecosystem, there are two alternative stable states corresponding to savanna (low tree cover), forest (high tree cover) respectively, and an unstable state

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(intermediate tree cover). The main factors driving force in determining the distribution of tree cover are the rainfall and fires, and the bistability here is due the positive feedback of fire interaction [15,16].

According to the data collected form satellite in [16], when the mean annual rainfall is less than 1000 mm, fire is less important since water becomes the main factor of limiting the tree growth. In this situation, tree cover often maintains at a relatively low level which may be savanna state. When the mean annual rainfall is more than 2000 mm, fire will be less likely to happen because such heavy rain leads to forest state. Only when the mean annual rainfall is between 1000 mm and 2000 mm, fire plays a key role in differentiating low and high tree cover. It was shown in [19] that fire can restrict the tree cover while climate and other external environmental conditions hold lush forest. As long as fire can reduce tree cover, the grass biomass drops off rapidly as tree cover increases [13]. Thus fire mainly spread on the land with tree cover less than 40% [1]. Also fire does not destroy the saplings which will sprout after. So fire drives the tree recruitment as a positive feedback [12,20].

In [16] the authors proposed a system of three ordinary differential equations of grass, tree saplings, and adult trees which incorporates the fire feedback with a nonlinear response function depending on grass abundance. So in their model the tree cover is determined by the combined interaction of rainfall and fire. While some qualitative analysis was conducted in [16] in a formal way, here we use a more specific nonlinear response function to quantify the model. Moreover we rigorously show the existence of precisely three positive steady states, and we also obtain exact parameter ranges of bistable dynamics. We also show that there are no limit cycles in the system, hence oscillatory states cannot occur. Note that in another system of four equations of grass, savanna saplings, savanna trees and forest trees [17], time-periodic states were found for certain parameter values. The spatial effect to the model was also considered in [12].

Bistability and corresponding hysteresis phenomena have arisen in various ecological systems via different mechanisms [2,4]. A classical example is the spruce budworm outbreak [7], and a general description is provided in [9]. More recently bistability (or alternative stable states) has also been found in shallow lake dynamics [11], eutrophication of aquatic ecosystems [3], coral reef dynamics [6,8], and oyster population dynamics [5].

In Section 2 we describe the differential equation model in details, and in Section 3 we present our main mathematical results accompanied by numerical simulations. Some concluding remarks are given in Section 4.

2. Model

We consider an ordinary differential equation model proposed by [16]:

$$\begin{cases} G' = \mu S + \gamma T - \beta G T, \\ S' = \beta G T - \omega(G) S - \mu S, \\ T' = \omega(G) S - \gamma T. \end{cases} \tag{2.1}$$

Here G is the total area of grass, S is the total area of tree saplings and T is the total area of adult trees. In (2.1), μ and γ are the mortality rates of saplings and trees respectively. After their death, the areas of saplings and trees revert to the one for grass in proportion to the number of trees and saplings. Hence the grassland recruitment is in form of $\mu S + \gamma T$. On the other hand the grassland loses its area when the adult trees establish saplings on. It is assumed that the saplings can only be successfully established on the grassland proportional to adult trees with a rate β. The growth rate of trees is proportional to the number of saplings with a nonlinear requirement rate $\omega(G)$ depending on the grassland. Indeed a higher value of G induces a higher possibility of fire, that implies a lower tree recruitment $\omega(G)$; while a lower value of G reduces the probability of fire and it increases the tree recruitment. Hence the fire feedback process is modeled by the nonlinear function $\omega(G)$. In [16], it is proposed that $\omega(G)$ is a sigmoidal function of grass

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