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Global quasi-neutral limit of Euler–Maxwell systems with velocity dissipation

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A B S T R A C T

We consider smooth solutions to the Cauchy problem for an isentropic Euler–Maxwell system with velocity dissipation and small physical parameters. For initial data uniformly close to constant equilibrium states, we prove the global-in-time convergence of the system as the parameters go to zero. The limit systems are the e-MHD system and the incompressible Euler equations, both with velocity dissipation. The proof of the results relies on a single uniform energy estimate with respect to the time and the parameters, together with compactness arguments. For this purpose, the classical energy estimates for the symmetrizable hyperbolic system are not sufficient. We construct a Lyapunov type energy by controlling the divergence and the curl of the velocity, the electric and magnetic fields.

1. Introduction

This paper concerns the global-in-time convergence of a one-fluid compressible Euler–Maxwell system with small parameters. The system arises in the theory of plasmas. The unknown functions are n, u, E and B standing for the scaled density, the velocity, the electric and magnetic fields, respectively. They depend on the time $t \geq 0$ and the position $x = (x_1, x_2, x_3) \in \mathbb{R}^3$. The physical parameters are the scaled Debye length $\lambda > 0$ (we refer to [4,8] for a precise description of this dimensionless parameter), the inverse of the speed of light $\nu > 0$ and the relaxation time $\tau > 0$.

We consider smooth solutions to the Cauchy problem in $\mathbb{R}^3$. The periodic problem can be dealt with in the same way. The scaled one-fluid Euler–Maxwell system reads (see [1,4,5,23])

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$$\begin{cases} \partial_t n + \operatorname{div}(nu) = 0, \\ \partial_t(nu) + \operatorname{div}(nu \otimes u) + \nabla p(n) = -n(E + \nu u \times B) - \dfrac{nu}{\tau}, \\ \lambda^2 \nu \partial_t E - \operatorname{curl} B = \nu n u, \quad \lambda^2 \operatorname{div} E = 1 - n, \\ \nu \partial_t B + \operatorname{curl} E = 0, \quad \operatorname{div} B = 0, \end{cases} \tag{1.1}$$

for $t > 0$ and $x \in \mathbb{R}^3$. Here p is the pressure function, supposed to be strictly increasing on $(0, +\infty)$. In this paper, we fix $\tau = 1$. We refer the reader to [25,35,36] for uniform energy estimates with respect to τ and the relaxation limit of the system as $\tau \to 0$. System (1.1) is complemented by the initial conditions depending on the parameters

$$(n, u, E, B)(t = 0) = (n_0^\varepsilon, u_0^\varepsilon, E_0^\varepsilon, B_0^\varepsilon), \tag{1.2}$$

where and in what follows we denote $\varepsilon = (\lambda, \nu)$ and we suppose

$$\lambda^2 \operatorname{div} E_0^\varepsilon = 1 - n_0^\varepsilon, \quad \operatorname{div} B_0^\varepsilon = 0. \tag{1.3}$$

For smooth solutions with $n > 0$, the momentum equation in (1.1) is equivalent to

$$\partial_t u + (u.\nabla)u + \nabla h(n) = -(E + \nu u \times B) - u, \tag{1.4}$$

where h is the enthalpy function, defined by

$$h'(n) = \frac{p'(n)}{n}.$$

Since p is strictly increasing on $(0, +\infty)$, so is h. The last term on the right-hand side of (1.4) represents the velocity dissipation in energy estimates. It plays a key role in our analysis. System (1.1) is symmetrizable hyperbolic when $n > 0$. According to the result of Kato (see [17,21]), it admits a unique local smooth solution when the initial data in $H^k(\mathbb{R}^3)$ with $k \geq 3$ being an integer. The solution is defined on a time interval $[0, T_\varepsilon]$ and stays in the standard space

$$\mathcal{C}\big([0, T_\varepsilon]; H^k(\mathbb{R}^3)\big) \cap \mathcal{C}^1\big([0, T_\varepsilon]; H^{k-1}(\mathbb{R}^3)\big),$$

where $T_\varepsilon > 0$ depends on the parameter ε. For simplicity, in what follows, we use the notation H^l, L^2 and L^∞ instead of $H^l(\mathbb{R}^3)$, $L^2(\mathbb{R}^3)$ and $L^\infty(\mathbb{R}^3)$, for all integer $l \geq 1$.

The physical parameters λ and ν are independent of each other. Since they are very small compared to the characteristic size of physical interest, it is important to study the limits of system (1.1) as these parameters go to zero. In the present paper we study two different asymptotic limits, which are the quasi-neutral limit $\lambda \to 0$ with $\nu = 1$ and the non-relativistic quasi-neutral limit $\varepsilon = (\lambda, \nu) \to 0$. In the second limit, we don't impose any relation between λ and ν.

The asymptotic analysis of (1.1) for smooth solutions is a well known problem. It starts from the local-in-time convergence of the system. See [31,32]. The non-relativistic limit $\nu \to 0$ and the quasi-neutral limit $\lambda \to 0$ were justified in [22] and [23], and the limit systems are the compressible Euler–Poisson system and the e-MHD system, respectively. The non-relativistic quasi-neutral limit $\varepsilon \to 0$ with a special relation $\nu = \lambda^2$ was justified in [24]. The result shows that the limit of (1.1) is the incompressible Euler equations. The zero-relaxation limit $\tau \to 0$ was studied in [9,25,36] and the limit system is the drift-diffusion equations. The results in the above papers are valid on uniform time intervals independent of the parameters. For relevant results for the local-in-time convergence of multi-dimensional Euler–Poisson systems, we refer the reader to [6,10,14,20,34,38] and references therein.

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