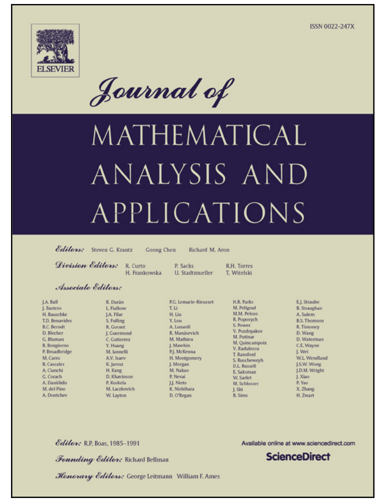

Decay properties of solutions to a 4-parameter family of wave equations

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DECAY PROPERTIES OF SOLUTIONS TO A 4-PARAMETER FAMILY OF WAVE EQUATIONS

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Abstract. In this paper, persistence properties of solutions are investigated for a 4-parameter family ($k-abc$ equation) of evolution equations having ($k+1$)-degree nonlinearities and containing as its integrable members the Camassa-Holm, the Degasperis-Procesi, Novikov and Fokas-Olver-Rosenau-Qiao equations. These properties will imply that strong solutions of the $k-abc$ equation will decay at infinity in the spatial variable provided that the initial data does. Furthermore, it is shown that the equation exhibits unique continuation for appropriate values of the parameters k, a, b, and c.

1. Introduction

For $k \in \mathbb{Z}^+$, $k \geq 2$ and $a, b, c \in \mathbb{R}$, we consider the Cauchy problem for the following $k-abc$ family of equations

$$u_t + u^k u_x - au^{k-2}u_x^3 + D^{-2}\partial_x \left[\frac{b}{k+1}u^{k+1} + cu^{k-1}u_x^2 - a(k-2)u^{k-3}u_x^4\right] + D^{-2}\left[[k(k+2) - 8a - b - c(k+1)]u^{k-2}u_x^3 - 3a(k-2)u^{k-3}u_x^3 u_{xx}\right] = 0 \tag{1.1}$$

where $D^{-2} \doteq (1-\partial_x^2)^{-1}$, and study its persistence properties. We show that if the initial data is endowed with exponential decay at infinity, then the corresponding solution will carry this property. When $a = 0$ and k a positive odd integer with $k \geq 1$ and $b \in [0, k(k+2)]$ then, utilizing the aforementioned behavior of strong solutions, we show that this equation exhibits unique continuation in the sense that if the initial value $u(x, 0)$ is given the property of decaying exponentially, then the solution $u(x,t)$ must be identically zero if assumed to be decaying exponentially at some later time $t > 0$. These are natural extensions of the results proved in Himonas, Misiołek, Ponce and Zhou [13] for the Camassa-Holm equation.

The $k-abc$ equation was first studied by Himonas and Mantzavinos [10] where they showed well-posedness in Sobolev spaces H^s for $s > 5/2$. They also provided a sharpness result on the data-to-solution map and proved that it is not uniformly continuous from any bounded subset of H^s into $C([0,T]; H^s)$. It was shown, however, that the solution map is Hölder continuous if H^s is equipped with a weaker H^r norm where $r \in [0, s)$. The equation was also studied in Barostichi, Himonas and Petronilho in [1] where they exhibited a power series method in abstract Banach spaces provided analytic initial data, thereby establishing a Cauchy-Kovalevsky type theorem for the $k-abc$ equation (1.1).

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