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Stable determination outside a cloaking region of two time-dependent coefficients in an hyperbolic equation from Dirichlet to Neumann map

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ABSTRACT

In this paper, we treat the inverse problem of determining two time-dependent coefficients appearing in a dissipative wave equation, from measured Neumann boundary observations. We establish in dimension $n \geq 2$ stability estimates with respect to the Dirichlet-to-Neumann map of these coefficients provided that they are known outside cloaking regions. Moreover, we prove that they can be stably recovered in larger subsets of the domain by enlarging the set of data.

1. Introduction

1.1. Statement of the problem

This paper deals with the inverse problem of determining two time-dependent coefficients in a dissipative wave equation from boundary observations. Let Ω be a bounded domain of $\mathbb{R}^n$, $n \geq 2$, with $\mathcal{C}^\infty$ boundary $\Gamma = \partial\Omega$. Given $T > 0$, we introduce the following dissipative wave equation

$$\begin{cases} \partial_t^2 u - \Delta u + a(x,t)\partial_t u + b(x,t)u = 0 & \text{in } Q = \Omega \times (0,T), \\ u(x,0) = u_0(x), \quad \partial_t u(x,0) = u_1(x) & \text{in } \Omega, \\ u(x,t) = f(x,t) & \text{on } \Sigma = \Gamma \times (0,T), \end{cases} \tag{1.1}$$

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where $f \in H^1(\Sigma)$, $u_0 \in H^1(\Omega)$, $u_1 \in L^2(\Omega)$, and the coefficients $a \in \mathcal{C}^2(Q)$ and $b \in \mathcal{C}^1(Q)$ are assumed to be real valued. It is well known (see [25] and Appendix B) that if $f(\cdot, 0) = u_{0|\Gamma}$, there exists a unique solution u to the equation (1.1) satisfying

$$u \in \mathcal{C}([0,T], H^1(\Omega)) \cap \mathcal{C}^1([0,T], L^2(\Omega)).$$

Moreover, there exists $C > 0$, such that

$$\|\partial_\nu u\|_{L^2(\Sigma)} + \|u(\cdot,t)\|_{H^1(\Omega)} + \|\partial_t u(\cdot,t)\|_{L^2(\Omega)} \leq C \left(\|f\|_{H^1(\Sigma)} + \|u_0\|_{H^1(\Omega)} + \|u_1\|_{L^2(\Omega)}\right). \tag{1.2}$$

Here ν denotes the unit outward normal to Γ at x and $\partial_\nu u$ stands for $\nabla u \cdot \nu$.

In the present paper, we address the uniqueness and the stability issues in the study of an inverse problem for the dissipative wave equation (1.1), in the presence of an absorbing coefficient a and a potential b that depend on both space and time variables. We consider three different sets of data and we aim to show that a and b can be recovered in some specific subsets of the domain, by probing it with disturbances generated on the boundary. The Dirichlet data f is considered as a disturbance that is used to probe the medium which is assumed to be quiet initially.

The problem of identifying coefficients appearing in hyperbolic boundary value problems was treated very well and there are many works that are relevant to this topic. In the case where the unknown coefficient is depending only on the spatial variable, Rakesh and Symes [26] proved by means of geometric optics solutions, a uniqueness result in recovering a time-independent potential in a wave equation from global Neumann data. The uniqueness by local Neumann data, was considered by Eskin [15] and Isakov [18]. In [6], Bellassoued, Choulli and Yamamoto proved a log-type stability estimate, in the case where the Neumann data are observed on any arbitrary subset of the boundary. Isakov and Sun [20] proved that the knowledge of local Dirichlet-to-Neumann map yields a stability result of Hölder type in determining a coefficient in a subdomain. As for the stability obtained from global Neumann data, one can see Sun [34], Cipolatti and Lopez [13]. The case of Riemannian manifold was considered by Bellassoued and Dos Santos Ferreira [7], Stefanov and Uhlmann [33]. In the case of Lorentzian manifolds Stefanov in [31] treats the weighted ray transform of integrating functions over lightlike geodesics. He proves support theorem if the Lorentzian manifold and the weight are analytic.

All the mentioned papers are concerned only with time-independent coefficients. In the case where the coefficient is also depending on the time variable, there is a uniqueness result proved by Ramm and Rakesh [27], in which they showed that a time-dependent coefficient, with a compact support, appearing in a wave equation with zero initial conditions, can be uniquely determined from the knowledge of global Neumann data, but only in a precise subset of the cylindrical domain Q that is made of lines making an angle of 45° with the t-axis and meeting the planes $t = 0$ and $t = T$ outside $\overline{Q}$. However, inspired by the work of [35], Isakov proved in [19], that the time-dependent coefficient can be recovered from the responses of the medium for all possible initial data, over the whole domain Q.

It is clear that with zero initial data, there is no hope to recover a time-dependent coefficient appearing in a hyperbolic equation over the whole cylindrical domain, even from the knowledge of global Neumann data, because the value of the solution can be effected by the value of the initial conditions, which is actually due to a fundamental concept concerning hyperbolic equations called the domain of dependence (see [16]). Moreover, we can prove that the backward light-cone with base Ω is a cloaking region, that is we can not uniquely recover the coefficients in this region.

As for uniqueness results, we have also the paper of Stefanov [30], in which he proved that a time-dependent potential, tempered in the time variable and appearing in a wave equation can be uniquely recovered from scattering data and the paper of Ramm and Sjöstrand [28], in which they proved a uniqueness result for a tempered time-dependent coefficient on an infinite time-space cylindrical domain $\Omega \times \mathbb{R}_t$.

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