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Periodic wave solutions and solitary wave solutions of generalized modified Boussinesq equation and evolution relationship between both solutions

Shaowei Li [a,c], Weiguo Zhang [a,b,*], Xiaoshuang Bu [b]

[a] *College of Management, University of Shanghai for Science and Technology, Shanghai, 200093, PR China*
[b] *College of Science, University of Shanghai for Science and Technology, Shanghai, 200093, PR China*
[c] *School of Mathematics & Information Engineering, Taizhou University, Taizhou, 317000, PR China*

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A B S T R A C T

In this paper, we focus on studying the exact solitary wave solutions and periodic wave solutions of the generalized modified Boussinesq equation $u_{tt} - \delta u_{ttxx} - (a_1 u + a_2 u^2 + a_3 u^3)_{xx} = 0$, as well as the evolution relationship between these solutions. Detailed qualitative analysis is conducted on traveling wave solutions of this equation, and global phase portraits in various parameter conditions are proposed. Various significant results about the existence of both solutions, including three forms of solitary wave solutions and four exact bounded periodic wave solutions in different conditions are obtained. Then, we further discuss the relationship between energy of Hamiltonian system corresponding to this equation and the periodic wave solutions and solitary wave solutions. It is concluded that the essential reason of periodic wave solutions and solitary wave solutions is the different values for the energy of Hamiltonian system corresponding to this equation. In addition, the limited relations of periodic wave solutions and solitary wave solutions with the energy of Hamiltonian system are proposed, and the schematic diagram of evolution from periodic wave solutions to solitary wave solutions is drawn.

1. Introduction

The nonlinear modified Boussinesq equation (called “IBq” for short)

$$u_{tt} - u_{xxtt} - u_{xx} + \frac{a}{2}(u^2)_{xx} = 0, \tag{1.1}$$

* Corresponding author.
E-mail addresses: lsw0576@yahoo.com (S. Li), zwgzwm@126.com (W. Zhang).

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is an important model equation in physics and hydromechanics, obtained from hydromechanics equations, it can be used to describe the spread of wave in magnetic field and replace "bad" Boussinesq equation [14] as follows

$$u_{tt} - u_{xxxx} - (u + u^2)_{xx} = 0. \tag{1.2}$$

Eq. (1.2) is a famous model proposed by Boussinesq in 1872 to describe shallow-water wave [1,3]. It can also be used to describe a series of physical phenomena about the spread of wave in plasma and nonlinear lattice [5,10,11]. The following variation of the modified Boussinesq equation (called "IMBq" for short)

$$u_{tt} - u_{xxtt} - u_{xx} + \frac{a}{3}(u^3)_{xx} = 0, \tag{1.3}$$

is always applied to the study on the properties of discordant lattice and the spread of nonlinear Alfvén wave [14]. Clarkson [4] has studied the Painlevé property of Eq. (1.1). Yang [19] has studied the solutions' blowup of boundary value problem for the following modified Boussinesq equation

$$u_{tt} - u_{xxtt} - (u + \sigma(u))_{xx} = 0, \tag{1.4}$$

when $\sigma(u) = au^p$. Using sine-cosine function method, Wazwaz [18] has studied the following generalized modified Boussinesq equation by power change

$$u_{tt} - (u + a(u^{2p})_{xx} - bu^p(u^p)_{xx})_{tt} = 0, \tag{1.5}$$

where p is a nonzero integer, assuming Eq. (1.5) has solutions in form

$$u(x,t) = \{\lambda cos^m(\mu\xi), \lambda\sin^m(2\mu\xi)\}, |\xi| \le \frac{\pi}{2\mu}, \tag{1.6}$$

where $\lambda, \mu, m \neq 0$ are undetermined parameters. Korsunsky [9] has listed the solitary solution to Eq. (1.1) as follows

$$u(x,t) = \frac{3(1-v^2)}{a} sech^2\frac{\sqrt{v^2-1}}{2v}(x-vt), v^2 > 1, \tag{1.7}$$

and the solitary wave solution to Eq. (1.3) as follows

$$u(x,t) = \pm\sqrt{\frac{6(1-v^2)}{a}} sech\frac{\sqrt{v^2-1}}{2v}(x-vt), v^2 > 1, a < 0. \tag{1.8}$$

Recently, Zhang [23] has studied the solitary wave solution in form

$$u(\xi) = \frac{Asech^2(\alpha\xi)}{B_0 + B_1 sech^2(\alpha\xi)}, \tag{1.9}$$

of the generalized modified Boussinesq equation

$$u_{tt} - \delta u_{xxtt} - (b_1 u + b_2 u^2 + b_3 u^3)_{xx} = 0, \delta > 0, \tag{1.10}$$

where A, B_0, B_1, α are undetermined constants. The above Formulas (1.7), (1.8) and (1.9) are all satisfied with that $u(\xi) \to 0$, as $|\xi| \to \infty$. They are the bell-shaped solitary wave solutions whose asymptotic value equals zero. Moreover, (1.7) and (1.8) are particular case of (1.9).

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