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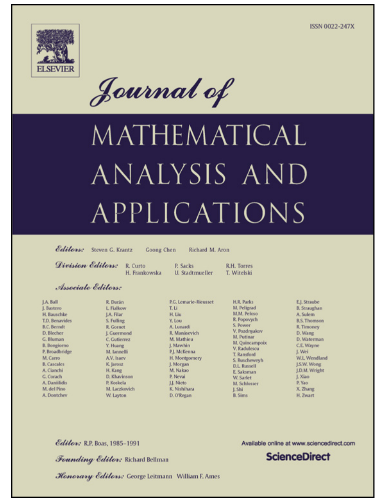

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Rosihan M. Ali, Roger W. Barnard, Alexander Yu. Solynin

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A NOTE ON BOHR'S PHENOMENON FOR POWER SERIES

ROSIHAN M. ALI, ROGER W. BARNARD, AND ALEXANDER YU. SOLYNIN

Abstract. Bohr's phenomenon, first introduced by Harald Bohr in 1914, deals with the largest radius r, $0 < r < 1$, such that the inequality $\sum_{k=0}^{\infty} |a_k| r^k \leq 1$ holds whenever the inequality $|\sum_{k=0}^{\infty} a_k z^k| \leq 1$ holds for all $|z| < 1$. The exact value of this largest radius known as *Bohr's radius*, which is $r_b = 1/3$, was discovered long ago. In this paper, we first discuss Bohr's phenomenon for the classes of even and odd analytic functions and for alternating series. Then we discuss Bohr's phenomenon for the class of analytic functions from the unit disk into the wedge domain $W_\alpha = \{w : |\arg w| < \pi\alpha/2\}$, $1 \leq \alpha \leq 2$. In particular, we find Bohr's radius for this class.

1. Introduction

Given the power series[1,2]

$$f(z) = \sum_{k=0}^{\infty} a_k z^k, \tag{1.1}$$

its *majorant series* is defined by

$$M_f(z) = \sum_{k=0}^{\infty} |a_k| r^k, \tag{1.2}$$

where and in the sequel, $r = |z|$. By basic complex analysis, the series (1.1) and (1.2) converge or diverge on open disks simultaneously, that is, for any given R, $0 < R \leq \infty$,

$$\left| \sum_{k=0}^{\infty} a_k z^k \right| < \infty \quad \text{for all } |z| < R$$

if and only if

$$\sum_{k=0}^{\infty} |a_k| r^k < \infty \quad \text{for all } r < R.$$

Of course, the values of functions $f(z)$ and $M_f(z)$ as well as values of certain norms of these functions may be very different. Our primary interest is the comparison of sup norms of $f(z)$ and $M_f(z)$ over the unit disk $\mathbb{D} = \{z : |z| < 1\}$ and disks with smaller radii: $\mathbb{D}_\rho = \{z : |z| < \rho\}$, $0 < \rho < 1$. This study was initiated by Harald Bohr in 1914 [11], who proved a weaker version of the following theorem.

Theorem 1.1. *If* $\left|\sum_{k=0}^{\infty} a_k z^k\right| \leq 1$ *in the unit disk* $\mathbb{D}$, *then* $\sum_{k=0}^{\infty} |a_k||z|^k \leq 1$ *in the disk* $\mathbb{D}_{1/3}$. *The radius* $\rho_b = 1/3$ *is the best possible.*

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[2]*Primary MSC (2010): 30B, 30C*