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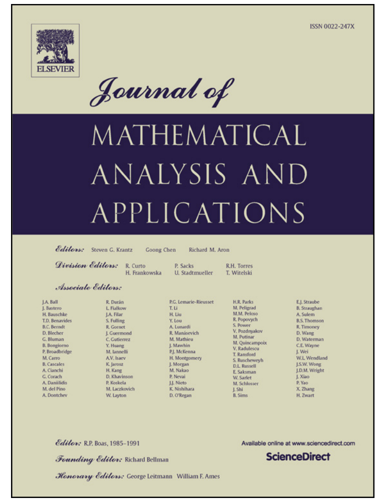

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An analytic expansion method for the valuation of double-barrier options under a stochastic volatility model

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Abstract

In this paper, we study a double-barrier option with a stochastic volatility model whose volatility is driven by a fast mean-reverting process, where the option's payoff is extinguished as the underlying asset crosses one of two barriers. By using an asymptotic analysis and Mellin transform techniques, we derive semi-analytic option pricing formulas with the sum of a leading-order term and a correction-order term, and then the accuracy of the first approximation price of the double-barrier option is verified by using Monte Carlo simulation. Moreover, we analyze the impact of stochastic volatility on the double-barrier option prices. Finally, we demonstrate that our results enhance the existing double-barrier option price structures in view of flexibility and applicability through the market price of volatility risk.

Keywords: Double-barrier option, Stochastic volatility, Asymptotic analysis, Mellin transform method

1. Introduction

A double-barrier option, which has a lower barrier and an upper barrier, is a combination of two independent knock-in or knock-out options whose payoff depends on its path throughout the lifetime of the option as well as on the underlying asset price at expiration rate. Once an underlying asset reaches either of these barriers, the knock-in option is immediately activated, but the knock-out option is quite the opposite, which is extinguished.

Double-barrier options have become popular derivatives in financial markets, which are the extended forms of ”single-barrier” options. One of the reasons barrier options have become very popular is that they are cheaper than standard vanilla options and, in fact, provide an equivalent level of protection when used as a hedge. The barrier option was first proposed by Merton [18] who gave the option pricing formula with a continuously monitored lower knock-out boundary, and then the formulae for various types of path-dependent options were derived by Goldman et al. [11]. Thereafter, Rich [21] and Rubinstein and Reiner [22] also used the probabilistic approaches under the equivalent martingale measure to deal with the pricing formula of European single-barrier options including the knock-in barrier calls and puts.

There are several papers that have already studied the pricing of double-barrier options using different methods. Kunitomo and Ikeda [17] obtained the probability density as an infinite sum of normal density functions for the underlying asset staying between two exponentially time-varying curved boundaries to price the analytic formula of double knock-out call and put options. Geman and Yor [10] used the inversion of the Laplace transform of the double-barrier option price to find the solution of the option prices numerically, and Carr and Lee [6] considered fast Fourier techniques (FFT) to calculate double-barrier derivatives. In addition, Buchen and Konstandatos [4] investigated the pricing of double-barrier options using the method of images.

The underlying asset pricing model of the double-barrier option mentioned above assumes that the return of the asset is normally distributed and that its variance is a constant. However, it is well known that the geometric

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