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Decay properties of solutions to the non-stationary magneto-hydrodynamic equations in half spaces

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L^1-decay properties of the strong solution (including the first and second order spacial derivatives) to the non-stationary magneto-hydrodynamic (MHD) equations are established in a half-space, and the weighted cases are also considered additionally.

1. Introduction and main results

Decay properties of viscous non-stationary magneto-hydrodynamic (MHD) equations are considered in the half-space $\mathbb{R}^n_+$ $(n \geq 2)$:

$$\begin{cases} \partial_t u - \dfrac{1}{Re}\Delta u + (u\cdot\nabla)u - S(B\cdot\nabla)B + \nabla\big(p + \dfrac{S}{2}|B|^2\big) = 0, \\ \partial_t B - \dfrac{1}{Rm}\Delta B + (u\cdot\nabla)B - (B\cdot\nabla)u = 0, \\ \nabla\cdot u = 0, \qquad \nabla\cdot B = 0, \end{cases}$$

where the unknown quantities $u = (u_1(x,t), \cdots, u_n(x,t))$, $B = (B_1(x,t), \cdots, B_n(x,t))$ and $p = p(x,t)$ denote the velocity of the fluid, the magnetic field and the pressure, respectively. The non-dimensional number Re is the Reynolds number, Rm is the magnetic Reynolds and $S = \frac{M^2}{ReRm}$ with M being the Hartman number. For simplicity of writing, let $Re = Rm = S = 1$, and p denotes the term $p + \frac{S}{2}|B|^2$. Then the initial boundary value problem of MHD equations can be written as follows:

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$$
\begin{cases}
\partial_t u - \Delta u + (u\cdot\nabla)u - (B\cdot\nabla)B + \nabla p = 0 & \text{in} \quad \mathbb{R}^n_+ \times (0,\infty), \\
\partial_t B - \Delta B + (u\cdot\nabla)B - (B\cdot\nabla)u = 0 & \text{in} \quad \mathbb{R}^n_+ \times (0,\infty), \\
\nabla\cdot u = 0, \quad \nabla\cdot B = 0 & \text{in} \quad \mathbb{R}^n_+ \times (0,\infty), \\
u(x,t) = B(x,t) = 0 & \text{on} \quad \partial\mathbb{R}^n_+ \times (0,\infty), \\
u(x,0) = a, \quad B(x,0) = b & \text{in} \quad \mathbb{R}^n_+,
\end{cases}
\tag{1.1}
$$

where $\mathbb{R}^n_+ = \{x = (x', x_n) \in \mathbb{R}^n \mid x_n > 0\}$ $(n \geq 2)$ is the upper-half space of $\mathbb{R}^n$; $u(x,0) = a(x)$ and $B(x,0) = b(x)$ are the initial velocity vector and magnetic field, respectively, which satisfy the compatibility condition in the sense of distribution: $\nabla\cdot a = \nabla\cdot b = 0$ in $\mathbb{R}^n_+$ and the normal components of a, b equal zero on $\partial\mathbb{R}^n_+$.

In this article, $C^\infty_{0,\sigma}(\mathbb{R}^n_+)$ denotes the set of all C^∞ real vector-valued functions $\varphi = (\varphi_1, \varphi_2, \cdots, \varphi_n)$ with compact support in $\mathbb{R}^n_+$, such that $\nabla\cdot\varphi = 0$ in $\mathbb{R}^n_+$. $L^r_\sigma(\mathbb{R}^n_+)$ $(1 < r < \infty)$ is the closure of $C^\infty_{0,\sigma}(\mathbb{R}^n_+)$ with respect to $\|\cdot\|_{L^r(\mathbb{R}^n_+)}$, where $L^r(\mathbb{R}^n_+)$ represents the usual Lebesgue space of vector-valued functions. For a given Banach space X, we denote the set of measurable functions L^r-integrable on $(0,T)$ with values in X and the set of functions continuing on $[0,T]$ with values in X by $C(0,T;X)$ and $L^r(0,T;X)$, respectively. Symbol C means a generic constant whose value may change from line to line.

Definition 1.1. Let $a, b \in L^2_\sigma(\mathbb{R}^n_+) \bigcap L^n(\mathbb{R}^n_+)$, $n \geq 2$. $(u,B) \in L^\infty(0,\infty; L^2_\sigma(\mathbb{R}^n_+))$ with $(\nabla u, \nabla B) \in L^2(0,\infty; L^2(\mathbb{R}^n_+))$ is called a strong solution of the MHD system (1.1) if

1) $u, B \in C(0,\infty; L^n_\sigma(\mathbb{R}^n_+))$;
2) (u,B) satisfies (1.1) in the sense of distribution in $\mathbb{R}^n_+ \times (0,\infty)$.

The following Helmholtz decomposition is valid ([6]):

$$
L^r(\mathbb{R}^n_+) = L^r_\sigma(\mathbb{R}^n_+) \oplus L^r_\pi(\mathbb{R}^n_+), \quad 1 < r < \infty,
$$

with

$$
L^r_\sigma(\mathbb{R}^n_+) = \{u = (u_1, u_2, \cdots, u_n) \in L^r(\mathbb{R}^n_+); \ \ \nabla\cdot u = 0, \ u_n|_{\partial\mathbb{R}^n_+} = 0\},
$$
$$
L^r_\pi(\mathbb{R}^n_+) = \{\nabla p \in L^r(\mathbb{R}^n_+); \ \ p \in L^r_{loc}(\overline{\mathbb{R}^n_+})\}.
$$

Let A denote the Stokes operator $-P\Delta$ in $\mathbb{R}^n_+$, where P is the associated bounded projection: $L^r(\mathbb{R}^n_+) \longrightarrow L^r_\sigma(\mathbb{R}^n_+)$, $1 < r < \infty$. Then (see [6]) the operator $-A$ generates a bounded analytic semigroup $\{e^{-tA}\}_{t\geq 0}$ in $L^r_\sigma(\mathbb{R}^n_+)$. So for each $a \in L^r_\sigma(\mathbb{R}^n_+)$, the function $v = e^{-tA}a$ is the unique solution of Stokes system in $L^r_\sigma(\mathbb{R}^n_+)$ with the corresponding function π, that is,

$$
\begin{cases}
\partial_t v - \Delta v + \nabla\pi = 0 & \text{in} \quad \mathbb{R}^n_+ \times (0,\infty), \\
\nabla\cdot v = 0 & \text{in} \quad \mathbb{R}^n_+ \times (0,\infty), \\
v(x,t) = 0 & \text{on} \quad \partial\mathbb{R}^n_+ \times (0,\infty), \\
v(x,0) = a & \text{in} \quad \mathbb{R}^n_+.
\end{cases}
$$

Bae [1,2] considered the Stokes flow $e^{-tA}a$, and established the L^r-decay estimates for $1 \leq r \leq \infty$ by imposing some constraint conditions on the initial datum a.

Now we state the first main result as follows, which concerns the weighted decay of the Stokes flow in $L^1(\mathbb{R}^n_+)$.

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