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On the outer pressure problem of the one-dimensional compressible Navier–Stokes equation with degenerate transport coefficients ☆

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A R T I C L E I N F O

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A B S T R A C T

This paper is concerned with the initial boundary value problem for the compressible Navier–Stokes system with degenerate transport coefficients. We prove the global existence of large solutions for the outer pressure problem of a viscous, heat-conducting, one-dimensional real gas. The proof is based on several detailed analysis and key a priori estimates on the bounds on the density and temperature.

1. Introduction

The one-dimensional compressible Navier–Stokes system in the Lagrangian coordinates can be written as:

$$\begin{cases} v_t - u_x = 0 \\ u_t - \sigma_x = 0 \\ (e + \frac{u^2}{2})_t - (\sigma u)_x + q_x = 0. \end{cases} \tag{1.1}$$

And the second law of thermodynamics is expressed by the Clausius–Duhem inequality

$$\eta_t + \left(\frac{q}{\theta}\right)_x \geq 0. \tag{1.2}$$

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Here v, u, σ, e, q, η and θ denote specific volume, velocity, stress, internal energy, heat flux, entropy and temperature respectively.

In the present paper, we consider the system (1.1) in the region $\{(x,t)|x \in I = [0,1], t \geq 0\}$ with the initial condition:

$$(v(x,0), u(x,0), \theta(x,0)) = (v_0(x), u_0(x), \theta_0(x)) \quad \text{on } I, \tag{1.3}$$

and the boundary conditions:

$$\begin{cases} \sigma(0,t) = \sigma(1,t) = -Q(t) < 0 \\ q(0,t) = q(1,t) = 0. \end{cases} \tag{1.4}$$

Here the outer pressure $Q(t) \in C^1([0,+\infty))$ is a given function.

We will consider a more realistic model than a real gas:

$$\sigma(v,\theta,u_x) = -p(v,\theta) + \frac{\mu u_x}{v} \tag{1.5}$$

satisfying Fourier's law of heat flux

$$q(v,\theta,\theta_x) = -\frac{\kappa\theta_x}{v}, \tag{1.6}$$

whose internal energy e and pressure p are coupled by the standard thermodynamical relation

$$e_v(v,\theta) = -p(v,\theta) + \theta p_\theta(v,\theta) \tag{1.7}$$

to comply with (1.3).

There are many progresses on the investigation of the one-dimensional compressible Navier–Stokes equations, cf. [2–9,15,11,12,16]. For the ideal gas and the constant coefficients case, we refer to [8,9,11] for the global existence and the asymptotic behavior of smooth solutions. For the coefficients dependent on the density and temperature case, the existence of global solutions has been investigated by many recent researchers, cf. [3,4,7,8,15,12] and the references cited therein. These studies indicate that temperature dependence on the viscosity μ is especially challenging but one can incorporate various forms of density dependence on μ and also temperature dependence on the conductivity κ. For results in this direction, Dafermos [3] and Dafermos and Hsiao [4] considered certain classes of solid-like materials in which μ and/or κ depend on density and where κ may depend on temperature. However, the latter is assumed to be bounded as well as uniformly bounded away from zero. Kawohl [8], Luo [15] and Jiang [5–7] considered a gas model that incorporates real-gas effects that occur in high-temperature regimes. For example, some of the assumptions on e and κ in [5–8,15] are that there are positive constants ν, $N(\underline{v})$, κ_0 and $\kappa_1(\underline{v})$ with $\underline{v} > 0$ such that for $v \geq \underline{v}$ and $\theta \geq 0$,

$$\begin{cases} e(v,0) \geq 0, \quad \nu(1+\theta^r) \leq e_\theta(v,\theta) \leq N(\underline{v})(1+\theta^r), \\ \kappa_0(1+\theta^q) \leq \kappa(v,\theta) \leq \kappa_1(\underline{v})(1+\theta^q), \end{cases} \tag{1.8}$$

where r and q are some constants satisfying $0 \leq r \leq 1$, $q \geq 2+2r$ in [8,15] and $q \geq 1+r$ in [5–7].

It is pointed out that the state functions e, p, μ, and κ usually depend on both v and θ, and in particular, the internal energy e grows as θ^{1+r} with $r \approx 0.5$, the conductivity κ grows as θ^q with $4.5 \leq q \leq 5.5$, and the viscosity μ increases like θ^p with $0.5 \leq p \leq 0.8$; see [1,16]. To incorporate these effects in the model, the state functions should be allowed to have a certain growth behavior. For this purpose, we make the following assumptions.

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