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Approximate representations of solutions to SVIEs, and an application to numerical analysis ☆

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A B S T R A C T

In this paper, we establish approximate representations of solutions to stochastic Volterra integral equations (SVIEs, for short), by virtue of solutions to a family of stochastic differential equations. As an application, we present two algorithms for solving SVIEs: stochastic theta method and splitting method, and prove the global 1/2 order convergence rates in L^2 norm.

1. Introduction

In this paper, we consider the following stochastic Volterra integral equation (SVIE, for short):

$$x(t) = \varphi(t) + \int_0^t b(t, s, x(s))ds + \int_0^t \sigma(t, s, x(s))dW(s),\ t \in [0, T]. \tag{1.1}$$

Here $W(\cdot)$ is a 1-dimensional Wiener process defined on some probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ with $\mathbb{F} = \{\mathcal{F}_t, t \in [0, T]\}$, and $\varphi : [0, T] \times \Omega \to \mathbb{R}^d$, b, $\sigma : [0, T]^2 \times \mathbb{R}^d \to \mathbb{R}^d$ are Borel measurable functions.

SVIEs generalize both stochastic differential equations (SDEs, for short) and deterministic integral equations. The theory of SDEs, being an important topic in stochastic processes, has been well investigated (see, for example, [3,4]). As natural and nontrivial extensions of SDEs, SVIEs own distinctive features. The main one is that SVIEs contain memories, which is closer to the reality. SVIEs also own interesting applications such as in stochastic control (see, [2,16]).

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On the other hand, SVIEs can be viewed as extensions of Volterra integral equations. Volterra integral/integro-differential equations arise in many applications such as elasticity, seismology, fluid flow, chemical reactions. In these applications, as problems with delay perturbation, integral terms are introduced. We refer the readers to [10] for more information on both theory and applications. In practice, noise sources are inevitable so that stochastic Volterra integral/integro-differential equations appears naturally.

SVIEs with regular kernels and driven by Brownian motions were first studied in the 1970–80s (see, for example, [1,7]). Later, Protter [13] studied SVIEs driven by general semimartingales. Using the Skorohod integral, Pardoux and Protter [12] also investigated SVIEs with anticipating coefficients.

However, in actual problems, it is not easy to obtain the exact solutions to SVIEs. Therefore, it is important, from a theoretical point of view, and even more for the sake of applications, to find their numerical solutions in an explicit form. Up to now, compared with algorithms for SDEs (see, for example, [9,11]), or deterministic Volterra equations (see, for example, [10]), algorithms for SVIEs are relatively less (see, [8,15,18]).

The main aim of the present paper is to establish approximate representations of solutions to SVIEs by virtue of solutions to SDEs. As an application of the representation result, we construct two algorithms for solving the numerical solutions to SVIEs: the stochastic theta method and the splitting method, and present their convergence rates, respectively.

This paper is organized as follows. In Section 2, we review the wellposedness results of SDEs and SVIEs, and introduce the main assumptions. In Section 3, we obtain our main result: presentation of solution to SVIE (1.1) by a family of solutions to SDEs. As an application, we introduce two numerical schemes for SVIE (1.1): the stochastic theta method, the splitting method in Subsection 4.1, Subsection 4.2, respectively; and obtain the global 1/2 order rates of convergence in the sense of the L^2 norm. At last, we present a computational experiment in Section 5.

2. Preliminaries

Recall that $\mathbb{R}^n$ is the n-dimensional Euclidean space with the standard Euclidean norm $|\cdot|$. We will denote by $L^p_{\mathbb{F}}(\Omega; L^q(0,T;\mathbb{R}^n))$ $(1 \le p,\ q < \infty)$ the Banach space consisting of all $\mathbb{R}^n$-valued $\mathbb{F}$-progressively measurable processes $\varphi(\cdot)$ such that $\mathbb{E}\Big(\int_0^T |\varphi(t)|^q dt\Big)^{\frac{p}{q}} < \infty$, with the canonical norm. When $p = q$, we write $L^p_{\mathbb{F}}(\Omega \times (0,T);\mathbb{R}^n)$ for simplicity.

The following lemma collects some standard results in SDE literature. We only list them.

Lemma 2.1. *Suppose that $b_0, \sigma_0 : \Omega \times [0,T] \times \mathbb{R}^d \to \mathbb{R}^d$ are $\mathbb{F}$-adapted stochastic fields, satisfying:*

(a) they are uniformly Lipschitz continuous with respect to $x \in \mathbb{R}^d$,
(b) $b_0(\cdot,0)$, $\sigma_0(\cdot,0) \in L^2_{\mathbb{F}}(\Omega \times (0,T);\mathbb{R}^d)$.

Then, for any $x \in \mathbb{R}^d$, the following SDE

$$X(t) = x + \int_0^t b_0(s,X(s))ds + \int_0^t \sigma_0(s,X(s))dW(s),\ t \in [0,T]$$

admits a unique solution. Furthermore, for any $p \ge 2$, the following estimate holds: