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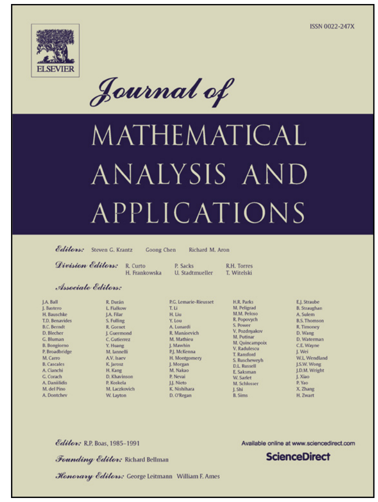


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# Regularization of an inverse nonlinear parabolic problem with time-dependent coefficient and locally Lipschitz source term

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**Abstract**

We consider a backward problem of finding a function $u$ satisfying a nonlinear parabolic equation in the form $u_t + a(t)Au(t) = f(t, u(t))$ subject to the final condition $u(T) = \varphi$. Here $A$ is a positive self-adjoint unbounded operator in a Hilbert space $H$ and $f$ satisfies a locally Lipschitz condition. This problem is ill-posed. Using quasi-reversibility method, we shall construct a regularized solution $u_\varepsilon$ from the measured data $a_\varepsilon$ and $\varphi_\varepsilon$. We show that the regularized problems are well-posed and that their solutions converge to the exact solutions. Error estimates of logarithmic type are given and a simple numerical example is presented to illustrate the method as well as verify the error estimates given in the theoretical parts.

**Keywords and phrases:** Nonlinear parabolic problem, Backward problem, Quasi-reversibility, Ill-posed problem, Contraction principle.
**Mathematics subject Classification 2000:** 35K05, 35K99, 47J06, 47H10.

## 1. Introduction

Let $(H, \|\cdot\|)$ be a Hilbert space with the inner product $\langle\cdot,\cdot\rangle$. Let $A$ be a positive self-adjoint operator defined on a dense subspace $D(A) \subset H$ such that $-A$ generates a compact contraction semi-group $S(t)$ on $H$. Let $f : [0, T] \times H \to H$ satisfy the locally Lipschitz condition: for each $M > 0$, there exists $k(M) > 0$ such that

$$\|f(t, u) - f(t, v)\| \leq k(M)\|u - v\| \text{ if } \max\{\|u\|, \|v\|\} \leq M. \tag{1}$$

We shall consider a backward problem of finding a function $u : [0, T] \to H$ such that

$$\begin{aligned} u_t + a(t)Au(t) &= f(t, u(t)), \quad 0 < t < T, \\ u(T) &= \varphi, \end{aligned} \tag{2}$$

where $a \in C([0, T])$ is a given real-valued function and $\varphi \in H$ is a prescribed final value.

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