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Luciano Abadias, Marta De León, José L. Torrea
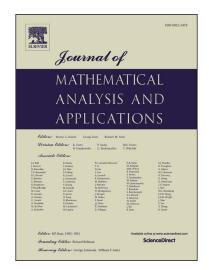
NON-LOCAL FRACTIONAL DERIVATIVES. DISCRETE AND CONTINUOUS

LUCIANO ABADIAS, MARTA DE LEÓN, AND JOSÉ L. TORREA

ABSTRACT. We prove maximum and comparison principles for the discrete fractional derivatives in the integers. Regularity results when the space is a mesh of length h, and approximation theorems to the continuous fractional derivatives are shown. When the functions are good enough (Hölder continuous), these approximation procedures give a measure of the order of approximation. These results also allow us to prove the coincidence, for Hölder continuous functions, of the Marchaud and Grünwald-Letnikov derivatives in every point and the speed of convergence to the Grünwald-Letnikov derivative. The discrete fractional derivative will be also described as a Neumann-Dirichlet operator defined by a semi-discrete extension problem. Some operators related to the Harmonic Analysis associated to the discrete derivative will be also considered, in particular their behavior in the Lebesgue spaces $\ell^p(\mathbb{Z})$.

1. INTRODUCTION

Fractional derivatives on time have been used to propose nonlocal models to describe non-diffusive transport in magnetically confined plasmas, see [10]. They also appear in the study of parabolic problems in which it is natural to take into account the past, see[2]. Some porous medium flow with fractional time derivative have been considered recently, see [3]. In these cases, regarding the fractional derivative on time, several discretization techniques play a crucial role. In the literature, we can find different representations of the classical definition of the Riemann-Liouville fractional derivative as Caputo, Marchaud or Grünwald-Letnikov, see [18]. In general they do not coincide, for example they have different behavior respect to constant functions.

In this paper we study discrete fractional derivatives. We shall prove maximum and comparison principles as well as regularity results. Also approximation theorems to the continuous fractional derivatives will be shown. When the functions are (Hölder continuous), these approximation procedures give a measure of the order of approximation. These results also allows us to prove the coincidence, for (Hölder continuous), functions, of the Marchaud and Grünwald-Letnikov derivatives in every point and the speed of convergence to the Grünwald-Letnikov derivative. The discrete fractional derivatives will be also described as a Neumann-Dirichlet operator defined by a semi-discrete extension problem. Some operators related to the Harmonic Analysis associated to the discrete derivative will be also considered, in particular their behavior in the Lebesgue spaces $\ell^p(\mathbb{Z})$.