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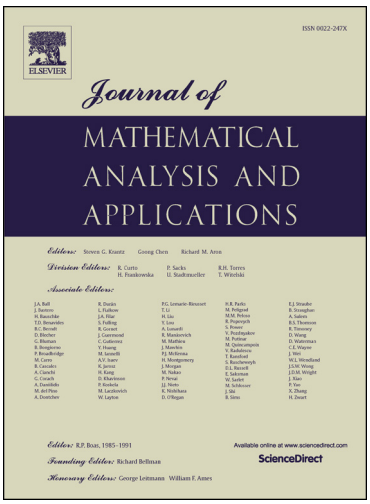

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A new blow-up condition for a parabolic equation with singular potential[1]

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Abstract

This paper is aimed at giving a new blow-up condition of the following parabolic problem with singular potential

$$|x|^{-s}u_t - \Delta u = |u|^{p-2}u,$$

which was studied extensively in recent years. By constructing a new functional, we obtain some blow-up conditions which include the possibility of the initial energy $J(u_0) \geq d$, where d is a positive constant which will be given in the main part of this paper. The results of this paper extend previous results where only the case $J(u_0) < d$ was considered.

Keywords: Parabolic equation with singular potential; Blow-up; Nehari manifold

1. Introduction

In this paper, we consider the following parabolic equation with a singular potential

$$\begin{cases} |x|^{-s}\dfrac{\partial u}{\partial t} - \Delta u = |u|^{p-2}u, & (x,t) \in \Omega \times (0,T), \\ u(x,t) = 0, & x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^N$ $(N > 2)$ with smooth boundary $\partial\Omega$, $u_0 \in H_0^1(\Omega)$, $T \in (0, \infty]$, $0 \leq s \leq 2$, $p \in (2, 2N/(N-2)]$.

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