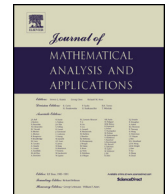




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Consistent pricing of VIX and equity derivatives with the 4/2 stochastic volatility plus jumps model

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ABSTRACT

In this paper, we develop a 4/2 stochastic volatility plus jumps model, namely, a new stochastic volatility model including the Heston model and 3/2 model as special cases. Our model is highly tractable by applying the Lie symmetries theory for PDEs, which means that the pricing procedure can be performed efficiently. In fact, we obtain a closed-form solution for the joint Fourier–Laplace transform so that equity and realized-variance derivatives can be priced. We also employ our model to consistently price equity and VIX derivatives. In this process, the quasi-closed-form solutions for future and option prices are derived. Furthermore, through adopting data on daily VIX future and option prices, we investigate our model along with the Heston model and 3/2 model and compare their different performance in practice. Our result illustrates that the 4/2 model with an instantaneous volatility of the form $(a\sqrt{V_t} + b/\sqrt{V_t})$ for some constants a, b presents considerable advantages in pricing VIX derivatives.

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1. Introduction

Since the Chicago Board Options Exchange (CBOE) launched the CBOE Volatility Index (VIX) futures in March 2004 and later VIX options in February 2006, the trading volume of derivatives on the VIX index has grown considerably over the last decade and become popular among investors. One reason is that VIX derivatives provide investors with a mechanism to directly and effectively invest in the volatility of the S&P500 index without having to factor in the price changes of the underlying instrument, dividends, interest rates or time to expiration. Moreover VIX derivatives are the first of an entire family of volatility products to be traded on exchanges. The index is also known as the “fear gauge” as in terms of market turmoil and large price movements, for the VIX index tends to rise, whereas when the market is easing

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upward in a long-run bull market, the VIX index remains low and steady. Naturally, this development has fueled the demand for models that are capable of simultaneously reproducing the observed characteristics of products on both indices, since derivatives products are traded on both the underlying index and the volatility index. Here, models that are able to capture these joint characteristics are known as consistent models.

A growing body of literature has emerged on the joint modeling of equity and VIX derivatives. One approach is adopted in Zhang and Zhu [22], Sepp [20], Zhu and Lian [23], Lian and Zhu [17] and Baldeaux and Badran [1]. The discounted price of derivatives can be expressed as a strict local martingale once the instantaneous dynamics of the underlying index are specified under the (putative) risk neutral probability measure $\mathbb{Q}$. Zhang and Zhu [22] derived an analytic formula for VIX futures under the assumption that the S&P500 is modeled by Heston [15]. Baldeaux and Badran [1] came up with more general formulae which allow for an empirical analysis to be performed to assess the appropriateness of the 3/2 framework the consistent pricing of equity and VIX derivatives. The Heston model [15] takes the instantaneous variance as a mean reverting squared Bessel process (usually called CIR or square root process since it displays a power 1/2 in the diffusion term), while the subsequent 3/2 model of Heston [14] and Platen [19] which is the inverse of a CIR process and, is still mean reverting with a power 3/2 in the diffusion term. Meanwhile, Grasselli [13] mentioned a less-known stochastic volatility model that combines as special cases the classic Heston with 3/2 model. This model considers as the superposition of the 1/2 and the 3/2 terms, i.e., we introduce an instantaneous volatility of the form $(a\sqrt{V_t} + b/\sqrt{V_t})$ for some constants a, b , where V_t is the CIR process. Although these authors provided exact solution by characteristic function approach for the price of VIX derivatives when the S&P500 is modeled by either Heston model or 3/2 model with simultaneous jumps in the underlying index, it remains to be shown whether 4/2 model is able to price the equity and VIX derivatives consistently. This paper aims to fill this vacuum.

Heston model has been justified a successful model in literature and in the banking industry for many reasons, e.g., smile and skew to be reproduced with parsimonious number of parameters, clear financial meaning on each parameter and its tractability. In addition, under a certain parameter restriction (the Feller condition), the volatility process remains strictly positive, which constitutes a nice property of the model. However, some shortcomings have been shown immediately in calibration of the model on real data. Feller condition is often violated because a high volatility-of-volatility parameter is required to fit the steep skews in equity markets. Moreover, when instantaneous volatility increases, the skew will flatten. Then, the Heston model assigns significant weight to very low and vanishing volatility scenarios and is unable to produce extreme paths with high volatility of volatility. Having said that, the Heston model still remains a good benchmark for any-single-factor stochastic volatility model that can be quickly calibrated on the market.

The selection of the inverse of a CIR model (3/2 model) for the underlying index is motivated by several observations in recent literatures. Compared with Heston model, both empirical and theoretical evidences suggest that the 3/2 model is a reasonable candidate for modeling instantaneous variance due to quick reversion when the process is high. Baldeaux and Badran [1] presented joint modeling of equity and VIX derivatives when the underlying index follows a 3/2 process with jumps in the index only. Following his conclusions, for 3/2 model, the implied volatility of VIX option are upward-sloping, which were consistent with market data. In fact, in the Heston model, the short-term skew flattens when the instantaneous variance increases, whereas in the 3/2 model, the short-term skew steepens when the instantaneous variance increases. Finally, applying to Fourier methodology, 3/2 model also remains to obtain an acceptable level of tractability when pricing derivatives products since closed form characteristic functions consist special functions like the hypergeometric confluent and Gamma functions.

There are two main contributions of this paper. Under the assumption that underlying index follows the 4/2 plus jumps model, one contribution is the derivation of closed-form solution for Fourier–Laplace transform of log-equity and realized-variance. The other is the derivation of quasi-closed-form solutions for

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