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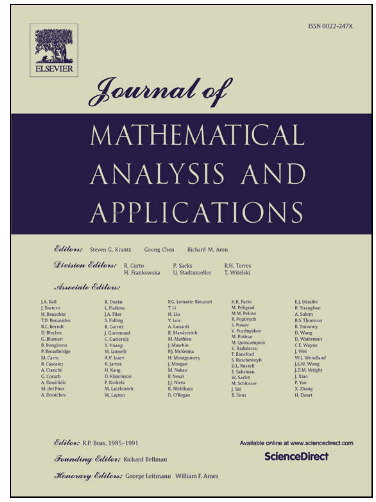

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REGULARITY OF SOLUTIONS OF ELLIPTIC PROBLEMS WITH A CURVED FRACTURE

S. ARICHE*, C. DE COSTER** AND S. NICAISE**

ABSTRACT. We consider the Laplace equation with a right-hand side concentrated on a curved fracture of class C^{m+2} for some non negative integer m (i.e., a sort of Dirac mass). We show that the solution belongs to a weighted Sobolev space of order m, the weight being the distance to this fracture. Our proof relies on a priori estimates in a dihedron or a cone with singularities for elliptic operators with variable coefficients. In both cases, such an estimate is obtained using a dyadic covering of the domain.

Key Words Laplace equation, Dirac measure, regularity.
AMS (MOS) subject classification 35R06, 35B65.

1. INTRODUCTION

We consider the boundary value problem

$$\begin{cases} -\Delta u = q\,\delta_\sigma, & \text{in} \quad \mathcal{O}, \\ u = 0, & \text{on} \quad \partial\mathcal{O}, \end{cases} \tag{1.1}$$

where $\mathcal{O}$ is a bounded domain of $\mathbb{R}^3$, the fracture σ is a one-dimensional curve strictly included in $\mathcal{O}$ of class C^{m+2} for some $m \in \mathbb{N}$ (see Figure 1) and q belongs to $L^2(\sigma)$.

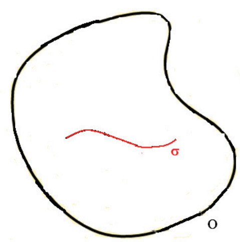

FIGURE 1. A model situation.

Such models are, for instance, used in fluid mechanics to save computational resources when the original system is too complex, see [15, 16]. Darcy's law in fractured domains is a typical example, where σ corresponds to a one-dimensional fracture. Some finite element method approximations of such problems can be found in [2, 3, 30] (in dimension two) and in [15, 16] (in dimension three).

The boundary value problem (1.1) is nonstandard because its solution cannot be in $\mathring{H}^1(\mathcal{O})$ since the right-hand side of this problem is not in its dual. Nevertheless it enters in the framework of problems set in spaces of distributions, that were studied in [5, 23, 28, 29], but in our case the datum belongs to $H^{-s}(\mathcal{O})$, for all $s > 1$, and therefore the shift theorem yields only a solution $u \in H^{2-s}(\mathcal{O})$, for all $s > 1$, that is not really satisfactory for numerical purposes. In [15], the author shows that a solution exists in a weighted Sobolev space, the weight being the distance to the fracture. More precisely [15, Corollary 2.2] shows that problem (1.1) has a weak solution $u \in \mathring{H}^1_\beta(\mathcal{O};\sigma)$ with $0 < \beta < 1$ (see Section 2 for the definition of this space), i.e., u is the unique function in $\mathring{H}^1_\beta(\mathcal{O};\sigma)$ such that

$$\int_{\mathcal{O}} \nabla u \cdot \nabla v = \int_\sigma q\gamma_\sigma v, \quad \forall v \in \mathring{H}^1_{-\beta}(\mathcal{O};\sigma), \tag{1.2}$$