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S. Ariche, C. De Coster, S. Nicaise

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## REGULARITY OF SOLUTIONS OF ELLIPTIC PROBLEMS WITH A CURVED FRACTURE

S. ARICHE\*, C. DE COSTER\*\* AND S. NICAISE\*\*

**ABSTRACT.** We consider the Laplace equation with a right-hand side concentrated on a curved fracture of class  $C^{m+2}$  for some non negative integer  $m$  (i.e., a sort of Dirac mass). We show that the solution belongs to a weighted Sobolev space of order  $m$ , the weight being the distance to this fracture. Our proof relies on a priori estimates in a dihedron or a cone with singularities for elliptic operators with variable coefficients. In both cases, such an estimate is obtained using a dyadic covering of the domain.

**Key Words** Laplace equation, Dirac measure, regularity.  
**AMS (MOS) subject classification** 35R06, 35B65.

### 1. INTRODUCTION

We consider the boundary value problem

$$(1.1) \quad \begin{cases} -\Delta u = q \delta_\sigma, & \text{in } \mathcal{O}, \\ u = 0, & \text{on } \partial\mathcal{O}, \end{cases}$$

where  $\mathcal{O}$  is a bounded domain of  $\mathbb{R}^3$ , the fracture  $\sigma$  is a one-dimensional curve strictly included in  $\mathcal{O}$  of class  $C^{m+2}$  for some  $m \in \mathbb{N}$  (see Figure 1) and  $q$  belongs to  $L^2(\sigma)$ .

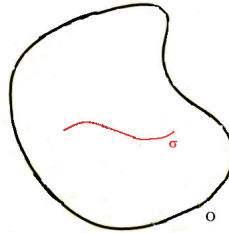


FIGURE 1. A model situation.

Such models are, for instance, used in fluid mechanics to save computational resources when the original system is too complex, see [15, 16]. Darcy's law in fractured domains is a typical example, where  $\sigma$  corresponds to a one-dimensional fracture. Some finite element method approximations of such problems can be found in [2, 3, 30] (in dimension two) and in [15, 16] (in dimension three).

The boundary value problem (1.1) is nonstandard because its solution cannot be in  $\dot{H}^1(\mathcal{O})$  since the right-hand side of this problem is not in its dual. Nevertheless it enters in the framework of problems set in spaces of distributions, that were studied in [5, 23, 28, 29], but in our case the datum belongs to  $H^{-s}(\mathcal{O})$ , for all  $s > 1$ , and therefore the shift theorem yields only a solution  $u \in H^{2-s}(\mathcal{O})$ , for all  $s > 1$ , that is not really satisfactory for numerical purposes. In [15], the author shows that a solution exists in a weighted Sobolev space, the weight being the distance to the fracture. More precisely [15, Corollary 2.2] shows that problem (1.1) has a weak solution  $u \in \dot{H}_\beta^1(\mathcal{O}; \sigma)$  with  $0 < \beta < 1$  (see Section 2 for the definition of this space), i.e.,  $u$  is the unique function in  $\dot{H}_\beta^1(\mathcal{O}; \sigma)$  such that

$$(1.2) \quad \int_{\mathcal{O}} \nabla u \cdot \nabla v = \int_{\sigma} q \gamma_\sigma v, \quad \forall v \in \dot{H}_{-\beta}^1(\mathcal{O}; \sigma),$$

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