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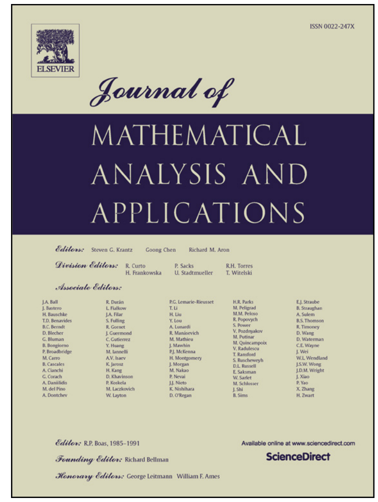

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Computation of Lyapunov Functions for Nonautonomous Systems on Finite Time-Intervals by Linear Programming

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Abstract

We present an algorithm for numerically computing Lyapunov functions for nonautonomous systems on finite time-intervals. The algorithm relies on a linear optimization problem and delivers a continuous and piecewise affine function on a compact set. The level-sets of such a Lyapunov function give concrete bounds on the time-evolution of the system on the time-interval and for time-periodic systems they deliver an ultimate bound on solutions. Four examples of computed finite-time Lyapunov functions are given.

Keywords: Lyapunov function, Finite-time Lyapunov function, Periodic-time system, Linear programming

1. Introduction

In the usual setting for a nonautonomous and continuous-time system, given by the differential equation $\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x})$, one studies the stability properties of the zero solution. Traditionally concepts such as uniform asymptotic stability or uniform ultimate boundedness, also known as practical stability, are studied in classical textbooks [15, 17, 24, 23]. Both of these, together with numerous other stability concepts, can be characterized by the existence of so-called Lyapunov functions. Lyapunov functions are real-valued functions from the state-space that are nonincreasing along the system's trajectories. They are fundamental tools when studying the qualitative behaviour of dynamical systems. Note that for nonlinear systems it is generally a very hard problem to identify or compute Lyapunov functions and for nonautonomous nonlinear systems even the linear case, i.e. $\dot{\mathbf{x}} = A(t)\mathbf{x}$, is quite involved.

Recently in [12] the concept of a finite-time Lyapunov function for nonautonomous systems on finite time intervals was introduced. The existence of such a function was shown to be equivalent to solutions being attracted to the zero solution on the interval in an appropriate norm. A different and much less strict approach was followed earlier in [13]

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