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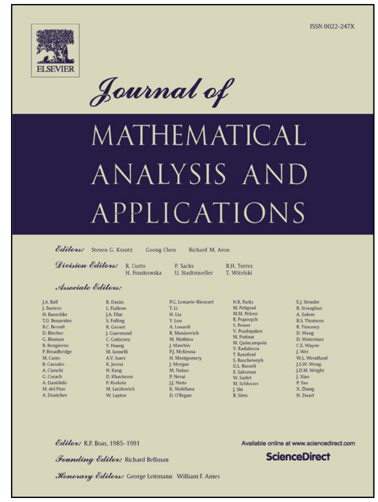

Geometry of reproducing kernels in model spaces near the boundary

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GEOMETRY OF REPRODUCING KERNELS IN MODEL SPACES NEAR THE BOUNDARY

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ABSTRACT. We study two geometric properties of reproducing kernels in model spaces K_θ where θ is an inner function: overcompleteness and existence of uniformly minimal systems of reproducing kernels which do not contain Riesz basic sequences. Both of these properties are related to the notion of the Ahern–Clark point. It is shown that "uniformly minimal non-Riesz" sequences of reproducing kernels exist near each Ahern–Clark point which is not an analyticity point for θ, while overcompleteness may occur only near the Ahern–Clark points of infinite order and is equivalent to a "zero localization property". In this context the notion of quasi-analyticity appears naturally, and as a by-product of our results we give conditions in the spirit of Ahern–Clark for the restriction of a model space to a radius to be a class of quasi-analyticity.

1. INTRODUCTION AND MAIN RESULTS

Let $H^2 = H^2(\mathbb{D})$ denote the standard Hardy space in the unit disk $\mathbb{D}$, and let θ be an inner function in $\mathbb{D}$. The *model* (or *star-invariant*) *subspace* K_θ of H^2 is then defined as

$$K_\theta = H^2 \ominus \theta H^2.$$

According to the famous Beurling theorem, any closed subspace of H^2 invariant with respect to the backward shift in H^2 is of the form K_θ. For the numerous applications of model spaces in operator theory and in operator-related complex analysis see [30, 31].

Recall that the function

$$k_\lambda(z) = k_\lambda^\theta(z) = \frac{1 - \overline{\theta(\lambda)}\theta(z)}{1 - \overline{\lambda} z}$$

is the *reproducing kernel* for the space K_θ corresponding to the point $\lambda \in \mathbb{D}$, that is, $(f, k_\lambda^\theta) = f(\lambda)$ for any function $f \in K_\theta$. We usually omit the index θ when it is clear from the context which model space we consider. In what follows we denote by $\widetilde{k}_\lambda$ the normalized reproducing kernel, that is, $\widetilde{k}_\lambda = k_\lambda / \|k_\lambda\|_2$.

Geometric properties of systems of reproducing kernels in model spaces is a deep and important subject which is studied extensively, see [22, 6, 28, 29, 7, 8, 9] for the study of completeness and [24, 21, 5, 11, 12, 10] for the results about bases of reproducing kernels. The main reason for that is that the geometric properties of reproducing kernels in a

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