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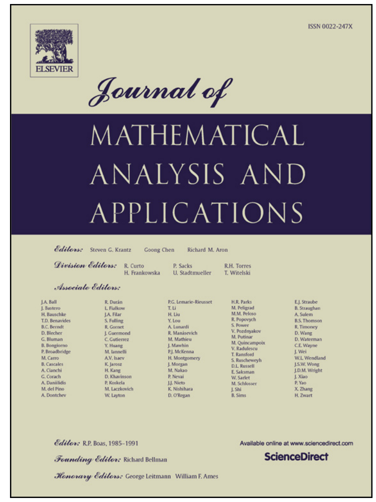

Vortex knots for the spheromak fluid flow and their moduli spaces

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Vortex knots for the spheromak fluid flow and their moduli spaces

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Abstract New exact solutions to the Euler hydrodynamics equations are constructed. A method for the study of vortex knots is developed for a special class of ideal fluid flows - the axisymmetric ones satisfying the Beltrami equation $\operatorname{curl}\mathbf{V}(\mathbf{x}) = \lambda\mathbf{V}(\mathbf{x})$. The method is based on a construction of the moduli spaces of vortex knots $\mathcal{S}(\mathbb{R}^3)$. Applying the method to the spheromak fluid flow we demonstrate that only those torus knots $K_{p,q}$ are realised as vortex knots for which p/q belongs to the interval $I_1 : \;\; 0.5 < \tau < M_1 \approx 0.8252$. We prove that each torus knot $K_{p,q}$ with $1/2 < p/q < 1/\sqrt{2}$ is realized on countably many invariant tori $\mathbb{T}^2 \subset \mathbb{R}^3$, while torus knots with $1/\sqrt{2} < p/q < M_1$ are realized only on finitely many tori. The moduli spaces of vortex knots $\mathcal{S}_m(\mathbb{B}^3_a)$ $(m = 1, 2, \cdots)$ are constructed for the spheromak fluid flows inside a ball $\mathbb{B}^3_a$ of radius a.

Keywords Euler equations · Exact solutions · Bessel functions · Plasma equilibria · Beltrami equation · Alexander polinomial

1 Introduction

I. We study exact solutions to the Euler equations of dynamics of an ideal incompressible fluid

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot \operatorname{grad})\mathbf{V} = -\frac{1}{\rho}\operatorname{grad} p, \qquad \operatorname{div}\mathbf{V} = 0 \tag{1.1}$$