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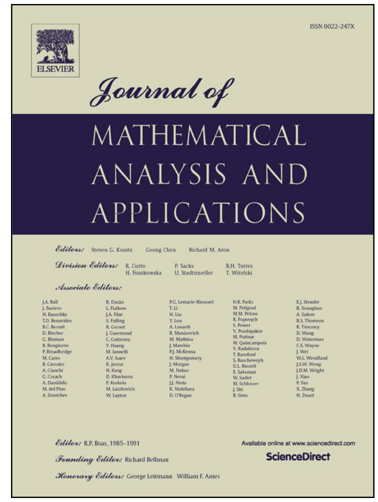

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ESSENTIAL NORMS OF INTEGRATION OPERATORS ON WEIGHTED BERGMAN SPACES

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ABSTRACT. We consider Volterra-type integration operators

$$T_g f(z) = \int_0^z f(\zeta) g'(\zeta)\, d\zeta$$

between Bergman spaces A^p_ω induced by weights satisfying a doubling property on the unit disc. We derive estimates for the operator norms and essential norms of $T_g \colon A^p_\omega \to A^q_\omega$ for $0 < p \le q < \infty$ extending earlier results by Peláez and Rättyä among others.

1. INTRODUCTION

Let $\mathbb{D}$ be the unit disc in the complex plane and $H(\mathbb{D})$ the algebra of all analytic functions in $\mathbb{D}$. For a fixed $g \in H(\mathbb{D})$, we consider the generalized Volterra integration operator T_g defined by

$$T_g f(z) = \int_0^z f(\zeta) g'(\zeta)\, d\zeta, \qquad z \in \mathbb{D},$$

for $f \in H(\mathbb{D})$. A systematic study of such operators acting on various spaces of analytic functions was initiated in the mid-1990s; see e.g. the survey articles [2] and [15] for background and references.

For $0 < p < \infty$ and a weight ω (i.e. a positive and integrable function in $\mathbb{D}$), let A^p_ω be the weighted Bergman space that consists of those $f \in H(\mathbb{D})$ for which the norm (or quasinorm if $0 < p < 1$)

$$\|f\|_{A^p_\omega} = \left(\int_{\mathbb{D}} |f|^p \omega\, dA \right)^{1/p}$$

is finite, where A is the normalized Lebesgue area measure on $\mathbb{D}$. In the context of such spaces, the operator T_g was first studied by Aleman and Siskakis in [4]. They proved, in particular, that for $1 \le p < \infty$ and a class of regular radial weights including the standard ones $\omega(z) = (1 - |z|)^\alpha$ with $\alpha > -1$, the operator T_g is bounded if and only if g belongs to the classical Bloch space, and compact if and only if g belongs to the little Bloch space. These results were extended to a more general setting of Bekollé-Bonami weights by Aleman and Constantin in [3]. In another direction, Peláez and Rättyä

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