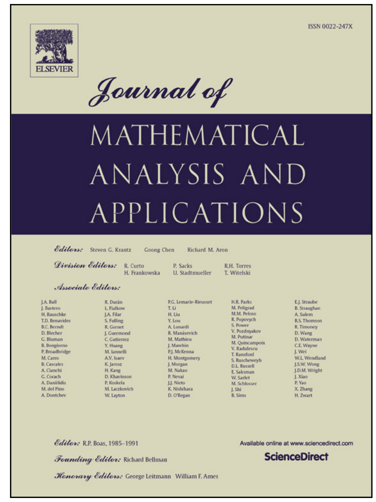

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R. Labbas, S. Maingot, D. Manceau, A. Thorel

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On the regularity of a generalized diffusion problem arising in population dynamics set in a cylindrical domain

R. Labbas, S. Maingot, D. Manceau & A. Thorel*

Normandie Univ, UNIHAVRE, LMAH, FR-CNRS-3335, 76600 Le Havre, France

Abstract

In this paper, we consider a generalized diffusion problem arising in population dynamics. To this end, we study a fourth order operational equation of elliptic type, with various boundary conditions. We show existence, uniqueness and regularity of a classical solution on a cylindrical domain under some necessary and sufficient conditions on the data. This elliptic problem is solved in $L^p(a,b;X)$, $p \in (1,+\infty)$, where $(a,b) \subset \mathbb{R}$ and X is a UMD Banach space. Our techniques use essentially the functional calculus and the semigroup theory.

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1 Introduction

Many problems in biology may be described by partial differential equations. It means that the model is constructed by averaging the density of a population and keeping only time and space variables. This leads to study the population density, so-called population dynamics, governed by reaction-diffusion equations. The model considered in general is the following:

$$\partial_t u = \alpha \Delta u + f(u), \quad \text{in } \mathbb{R}_+ \times \Omega, \tag{1.1}$$

where $u(t,x)$ is the density of population at time t and position x, Ω is an open set of $\mathbb{R}^d$, $d \geq 2$, and f is a non-linear growth interaction. The diffusion operator term Δu is obtained from Fick's law. In 1981, D.S. Cohen and J.D. Murray [6] derived a more complete model with a biharmonic term added to the harmonic one. This model has been obtained by studying the motility of cells, for which they noticed that the classical diffusion model was not sufficient. This model is:

$$\partial_t u = k_1 \Delta u - k_2 \Delta^2 u + f(u) \quad \text{in } \mathbb{R}_+ \times \Omega, \tag{1.2}$$

where $k_1, k_2 \in \mathbb{R} \setminus \{0\}$. The biharmonic term $\Delta^2 u$ represents the long range diffusion, whereas the harmonic term Δu represents the short range diffusion. Another derivation of this model has been obtained in 1984 by F.L. Ochoa [20] who studied the dynamics of (1.2) when f is a cubic function. His work brings out the importance of the biharmonic term in the diffusion of the population.

The classical study of nonlinear problems like (1.1) or (1.2) needs first to consider the associated linearized steady problem. This first step is essential to deduce maximal regularity results for the linearized unsteady problem. From these maximal regularity results, we can obtain the existence and uniqueness of the nonlinear problem by a fixed point theorem. In the case where Ω is an open set with C^4-boundary, the result is well-known. For problem (1.1), we can find a detailed study in [22], see section 4, p. 13 for instance. For problem (1.2), results of existence and regularity are obtained in Hilbertian case in [19] and some other results were recently obtained in [5].

In this paper, we study problem (1.2) in the linear steady case on a bounded cylindrical open set $\Omega := (a,b) \times \omega$ of $\mathbb{R}^d$, with ω a bounded open set of $\mathbb{R}^{d-1}$ with C^2-boundary and $f \in L^p(\Omega)$, $p \in (1,+\infty)$, *i.e.*

$$k_2 \Delta^2 u - k_1 \Delta u = f \quad \text{on } \Omega. \tag{1.3}$$