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# Concentration-compactness principle for nonlocal scalar field equations with critical growth

João Marcos do Ó<sup>1,\*</sup>, Diego Ferraz<sup>1</sup>

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## Abstract

The aim of this paper is to study a concentration-compactness principle for homogeneous fractional Sobolev space  $\mathcal{D}^{s,2}(\mathbb{R}^N)$  for  $0 < s < \min\{1, N/2\}$ . As an application we establish Palais-Smale compactness for the Lagrangian associated to the fractional scalar field equation  $(-\Delta)^s u = f(x, u)$  for  $0 < s < 1$ . Moreover, using an analytic framework based on  $\mathcal{D}^{s,2}(\mathbb{R}^N)$ , we obtain the existence of ground state solutions for a wide class of nonlinearities in the critical growth range.

*Keywords:* Scalar field equation, Fractional Laplacian, Concentration-compactness  
*2008 MSC:* 35J60, 35J20, 47J30, 49J35, 35B33

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## 1. Introduction

The main goal of the present work is to analyze a concentration-compactness principle for homogeneous fractional Sobolev spaces. As an application, we address questions on compactness of the associated energy functional to the following nonlocal scalar field equation

$$(-\Delta)^s u = f(x, u) \quad \text{in } \mathbb{R}^N, \quad (\mathcal{P}_s)$$

where  $0 < s < 1$ , and the nonlinearity  $f(x, t)$  is supposed to be a *asymptotic self-similar function* (see Sect. 3.1 for the precise definition), which is a variation of the critical nonlinearity. Here  $(-\Delta)^s$  is the fractional Laplacian defined by the relation

$$\mathcal{F}((-\Delta)^s u)(\xi) = |\xi|^{2s} \mathcal{F}u(\xi), \quad \xi \in \mathbb{R}^N,$$

where  $\mathcal{F}u$  is the Fourier transform of  $u$ , i.e.

$$\mathcal{F}u(x) = \frac{1}{(2\pi)^{N/2}} \int_{\mathbb{R}^N} u(\xi) e^{-i\xi \cdot x} d\xi, \quad x \in \mathbb{R}^N. \quad (1.1)$$

Let  $\mathcal{S}$  be the Schwartz space consisting of rapidly decaying  $C^\infty$  functions in  $\mathbb{R}^N$  which, together with all their derivatives, vanish at the infinity faster than any power of  $|x|$ . Equivalently, if  $u \in \mathcal{S}$  the fractional Laplacian of  $u$  can be computed by the following singular integral

$$(-\Delta)^s u(x) = C(N, s) \lim_{\varepsilon \rightarrow 0^+} \int_{\mathbb{R}^N \setminus B_\varepsilon(0)} \frac{u(x) - u(y)}{|x - y|^{N+2s}} dy,$$

for a suitable positive normalizing constant

$$C(N, s) = \left( \int_{\mathbb{R}^N} \frac{1 - \cos \zeta_1}{|\zeta|^{N+2s}} d\zeta \right)^{-1}.$$

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