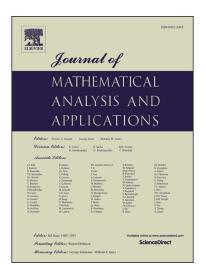
Accepted Manuscript

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 PII:
 S0022-247X(16)30849-6

 DOI:
 http://dx.doi.org/10.1016/j.jmaa.2016.12.053

 Reference:
 YJMAA 20993

To appear in: Journal of Mathematical Analysis and Applications

Received date: 15 July 2016

Please cite this article in press as: J.M. do Ó, D. Ferraz, Concentration-compactness principle for nonlocal scalar field equations with critical growth, *J. Math. Anal. Appl.* (2017), http://dx.doi.org/10.1016/j.jmaa.2016.12.053

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Concentration-compactness principle for nonlocal scalar field equations with critical growth

João Marcos do Ó^{1,*}, Diego Ferraz¹

Abstract

The aim of this paper is to study a concentration-compactness principle for homogeneous fractional Sobolev space $\mathcal{D}^{s,2}(\mathbb{R}^N)$ for $0 < s < \min\{1, N/2\}$. As an application we establish Palais-Smale compactness for the Lagrangian associated to the fractional scalar field equation $(-\Delta)^s u = f(x, u)$ for 0 < s < 1. Moreover, using an analytic framework based on $\mathcal{D}^{s,2}(\mathbb{R}^N)$, we obtain the existence of ground state solutions for a wide class of nonlinearities in the critical growth range.

Keywords: Scalar field equation, Fractional Laplacian, Concentration-compactness 2008 MSC: 35J60, 35J20, 47J30, 49J35, 35B33

1. Introduction

The main goal of the present work is to analyze a concentration-compactness principle for homogeneous fractional Sobolev spaces. As an application, we address questions on compactness of the associated energy functional to the following nonlocal scalar field equation

$$(-\Delta)^s u = f(x, u) \quad \text{in } \mathbb{R}^N,$$
 (\mathcal{P}_s)

where 0 < s < 1, and the nonlinearity f(x,t) is supposed to be a asymptotic self-similar function (see Sect. 3.1 for the precise definition), which is a variation of the critical nonlinearity. Here $(-\Delta)^s$ is the fractional Laplacian defined by the relation

$$\mathscr{F}\left((-\Delta)^{s}u\right)(\xi) = \left|\xi\right|^{2s} \mathscr{F}u(\xi), \ \xi \in \mathbb{R}^{N},$$

where $\mathscr{F}u$ is the Fourier transform of u, i.e.

$$\mathscr{F}u(x) = \frac{1}{(2\pi)^{N/2}} \int_{\mathbb{R}^N} u(\xi) e^{-i\xi \cdot x} \,\mathrm{d}\xi, \ x \in \mathbb{R}^N.$$
(1.1)

Let \mathscr{S} be the Schwartz space consisting of rapidly decaying C^{∞} functions in \mathbb{R}^N which, together with all their derivatives, vanish at the infinity faster than any power of |x|. Equivalently, if $u \in \mathscr{S}$ the fractional Laplacian of u can be computed by the following singular integral

$$(-\Delta)^s u(x) = C(N,s) \lim_{\varepsilon \to 0^+} \int_{\mathbb{R}^N \setminus B_\varepsilon(0)} \frac{u(x) - u(y)}{|x - y|^{N+2s}} \, \mathrm{d}y,$$

for a suitable positive normalizing constant

$$C(N,s) = \left(\int_{\mathbb{R}^N} \frac{1 - \cos \varsigma_1}{|\varsigma|^{N+2s}} \,\mathrm{d}\varsigma\right)^{-1}.$$

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