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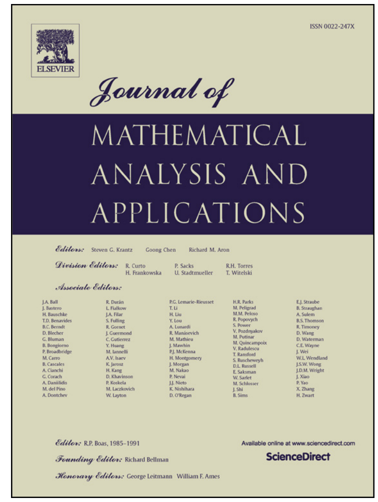

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Concentration-compactness principle for nonlocal scalar field equations with critical growth

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Abstract

The aim of this paper is to study a concentration-compactness principle for homogeneous fractional Sobolev space $\mathcal{D}^{s,2}(\mathbb{R}^N)$ for $0 < s < \min\{1, N/2\}$. As an application we establish Palais-Smale compactness for the Lagrangian associated to the fractional scalar field equation $(-\Delta)^s u = f(x,u)$ for $0 < s < 1$. Moreover, using an analytic framework based on $\mathcal{D}^{s,2}(\mathbb{R}^N)$, we obtain the existence of ground state solutions for a wide class of nonlinearities in the critical growth range.

Keywords: Scalar field equation, Fractional Laplacian, Concentration-compactness
2008 MSC: 35J60, 35J20, 47J30, 49J35, 35B33

1. Introduction

The main goal of the present work is to analyze a concentration-compactness principle for homogeneous fractional Sobolev spaces. As an application, we address questions on compactness of the associated energy functional to the following nonlocal scalar field equation

$$(-\Delta)^s u = f(x,u) \quad \text{in } \mathbb{R}^N, \tag{$\mathcal{P}_s$}$$

where $0 < s < 1$, and the nonlinearity $f(x,t)$ is supposed to be a *asymptotic self-similar function* (see Sect. 3.1 for the precise definition), which is a variation of the critical nonlinearity. Here $(-\Delta)^s$ is the fractional Laplacian defined by the relation

$$\mathscr{F}\left((-\Delta)^s u\right)(\xi) = |\xi|^{2s}\,\mathscr{F}u(\xi),\ \xi \in \mathbb{R}^N,$$

where $\mathscr{F}u$ is the Fourier transform of u, i.e.

$$\mathscr{F}u(x) = \frac{1}{(2\pi)^{N/2}}\int_{\mathbb{R}^N} u(\xi)e^{-i\xi\cdot x}\,\mathrm{d}\xi,\ x \in \mathbb{R}^N. \tag{1.1}$$

Let $\mathscr{S}$ be the Schwartz space consisting of rapidly decaying C^∞ functions in $\mathbb{R}^N$ which, together with all their derivatives, vanish at the infinity faster than any power of $|x|$. Equivalently, if $u \in \mathscr{S}$ the fractional Laplacian of u can be computed by the following singular integral

$$(-\Delta)^s u(x) = C(N,s)\lim_{\varepsilon\to 0^+}\int_{\mathbb{R}^N\setminus B_\varepsilon(0)} \frac{u(x)-u(y)}{|x-y|^{N+2s}}\,\mathrm{d}y,$$

for a suitable positive normalizing constant

$$C(N,s) = \left(\int_{\mathbb{R}^N}\frac{1-\cos\varsigma_1}{|\varsigma|^{N+2s}}\,\mathrm{d}\varsigma\right)^{-1}.$$

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