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Application of geometric calculus in numerical analysis and difference sequence spaces

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A B S T R A C T

The main purpose of this paper is to introduce the geometric difference sequence space $\ell_\infty^G(\Delta_G)$ and prove that $\ell_\infty^G(\Delta_G)$ is a Banach space with respect to the norm $\|.\|_{\Delta_G}^G$. Also we compute the α-dual, β-dual and γ-dual spaces. Finally we obtain the Geometric Newton–Gregory interpolation formulae.

1. Introduction and notations

In 1967 Robert Katz and Michael Grossman created the first system of non-Newtonian calculus, which we call the geometric calculus. In 1970 they had created an infinite family of non-Newtonian calculi, each of which differs markedly from the classical calculus of Newton and Leibniz. Among other things, each non-Newtonian calculus possesses four operators: a gradient (i.e. an average rate of change), a derivative, an average, and an integral. For each non-Newtonian calculus there is a characteristic class of functions having a constant derivative.

In view of pioneering work carried out in this area by Grossman and Katz [10] we will call this calculus as multiplicative calculus, although the term of exponential calculus can also be used. The operations of multiplicative calculus will be called as multiplicative derivative and multiplicative integral. We refer to Grossman and Katz [10], Stanley [18], Bashirov et al. [2,3], Grossman [9] for elements of multiplicative calculus and its applications. An extension of multiplicative calculus to functions of complex variables is handled by Bashirov and Rıza [1], Çakmak and Başar [5], Tekin and Başar [19], Türkmen and Başar [20], Uzer [22]. In [13], Kadak and Özlük studied the generalized Runge–Kutta method with respect to non-Newtonian calculus. Kadak and Efe [11] and Kadak et al. [12] studied certain new types of sequence

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spaces over the Non-Newtonian Complex Field. Çakmak and Başar [4] construct the field $\mathbb{C}_*$ of $*$-complex numbers and the concept of $*$-metric. Also they give the definitions and the basic important properties of $*$-boundedness and $*$-continuity. Later the space $\mathbb{C}_*(\Omega)$ of $*$-continuous functions which forms a vector space with respect to the non-Newtonian addition and scalar multiplication, and is a Banach space, is introduced. Finally, multiplicative calculus (MC) which is one of the most popular non-Newtonian calculi and created by the famous '*exp*' function, is applied to complex numbers and functions to investigate some advance inner product properties together with the inclusion relationship between $\mathbb{C}_*(\Omega)$ and the set of $\mathbb{C}'_*(\Omega)$-differentiable functions. The line and double integrals in the sense of non-Newtonian calculus ($*$-calculus) are given by Çakmak and Başar [6]. Moreover, in the sense of $*$-calculus, the fundamental theorem of calculus for line integrals and double integrals is stated with some applications. Based on multiplicative calculus matrix transformations in sequence spaces are studied and characterized by Çakmak and Başar [7]. Also, a brief introduction to $*$-summability based on multiplicative type addition (or just multiplication) is given and the multiplicative dual $*$-summability methods using $*$-Stieltjes integral and multiplicative differentiation under the $*$-integral sign are introduced. The problem of cylindrical wave incidence on a conducting half plane has been considered by Uzer [21]. A modal solution for Green's function of the problem is transformed into contour integral representations in a complex plane. Some contour deformations and changes of variables are then made for the integrals. The multiplicative calculus is employed in deriving an expression that can be used for obtaining approximate solutions when the observation angles are away from the RSBs of the conducting half plane. The derived expressions are seen to be very simple for implementing in any computational environment.

Geometric calculus is an alternative to the usual calculus of Newton and Leibniz. It provides differentiation and integration tools based on multiplication instead of addition. Every property in Newtonian calculus has an analog in multiplicative calculus. Generally speaking multiplicative calculus is a methodology that allows one to have a different look at problems which can be investigated via calculus. In some cases, for example for growth related problems, the use of multiplicative calculus is advocated instead of a traditional Newtonian one.

The main aim of this paper is to construct the difference sequence space $\ell_\infty^G(\Delta_G)$ over geometric real numbers which forms a Banach space with the norm defined on it and obtain the Geometric Newton–Gregory interpolation formulae which are more useful than Newton–Gregory interpolation formulae in the ordinary sense.

Before we establish new results, we recall the construction of arithmetics generated by different generators and the geometric arithmetic, which is the keyword of the whole article.

2. α-generator and geometric real field

A *generator* is a one-to-one function whose domain is the set of real numbers $\mathbb{R}$, and range is a set $B \subset \mathbb{R}$. For example, the identity function I and the exponential function *exp* are generators. We consider any generator α with realm i.e. domain, say, A and range B, by α-*arithmetic*, we mean the arithmetic whose operations and ordering relation are defined as follows:

$$\begin{array}{ll}
\alpha\text{-addition} & x\dot{+}y = \alpha[\alpha^{-1}(x) + \alpha^{-1}(y)] \\
\alpha\text{-subtraction} & x\dot{-}y = \alpha[\alpha^{-1}(x) - \alpha^{-1}(y)] \\
\alpha\text{-multiplication} & x\dot{\times}y = \alpha[\alpha^{-1}(x) \times \alpha^{-1}(y)] \\
\alpha\text{-division} & x\dot{/}y = \alpha[\alpha^{-1}(x)/\alpha^{-1}(y)] \\
\alpha\text{-order} & x\dot{<}y \Leftrightarrow \alpha^{-1}(x) < \alpha^{-1}(y)
\end{array}$$

for $x, y \in A$, where A is a domain of the function α.

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