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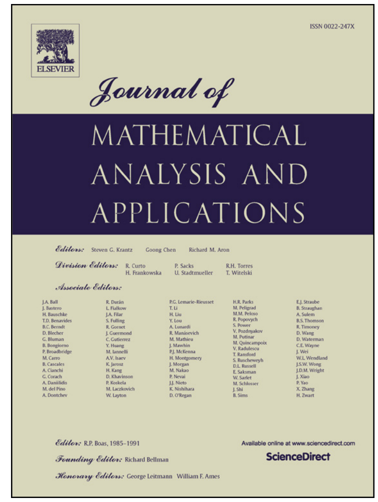

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On the simplification of the form of Lie transformation groups admitted by systems of evolution differential equations

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For a system of two partial differential equations in two independent variables the Lie point symmetry generator has the form $\tau(t,x,u,v)\partial_t+\xi(t,x,u,v)\partial_x+\eta(t,x,u,v)\partial_u+\mu(t,x,u,v)\partial_v$. We consider a system of evolution equations of general class and we provide restrictions on the functional forms of the coefficient functions of the Lie generator. These restrictions reduce the amount of calculations in determining the Lie symmetries of the system under study. We show that $\tau=\tau(t)$ except in one special case and we present further restrictions on the remaining coefficient functions.

Keywords: Nonlinear systems of evolution equations;Quasi-linear systems of evolution equations;Lie groups of transformations

1 Introduction

Lie symmetry methods are perhaps the most powerful tool currently available in the area of nonlinear partial differential equations (pdes). Lie symmetries are point transformations, which means that they are transformations in the space of the dependent and the independent variables of a pde. They form a continuous Lie group of transformations, which leave a pde (or a system of pdes) invariant. The main applications of Lie symmetries is to generate new solutions, directly using known solutions or via similarity reductions. A number of textbooks on Lie symmetries and their applications to differential equations exist in the literature. See for example, in [2, 4–6, 19, 24, 42, 43].

The theory of continuous Lie groups was developed by the Norwegian Mathematician Sophus Lie in the 19th century. He showed that such transformations can be expressed in terms of infinitesimal generators and he developed a method for constructing the infinitesimal generators admitted by a given pde (or a system of pdes). The coefficient functions of the Lie infinitesimal generator satisfy a system of pdes which is known as determining system. Each equation of this system is linear and homogeneous in the coefficient functions and their derivatives. In general, the determining system is overdetermined.

Although the method is easy to be applied, in some cases computing the Lie point symmetries of differential equations and especially for systems, it often appears to be a very difficult task. This is why it is important and useful to know some a-priori restrictions on the form of coefficients of Lie point symmetry generators depending on the structure of systems of differential equations under study.

There is a continuing interest in finding Lie symmetries for system of nonlinear differential equations, see for example in [3, 10–14, 20, 34, 38–40, 48]. Ibragimov [25–27] provides excellent sources of references for these results as well as for their many and varied physical applications.

There were a lot of attempts in computation of Lie symmetries of differential equations using different systems of computer algebras, such as MATHEMATICA, MAPLE, MACSYMA, REDUCE, AXIOM, MuPAD etc. Different symbolic manipulation packages [16, 17, 21, 50, 56] (see also detailed review in [22, 23]). These programs, although powerful, are not guarantee to