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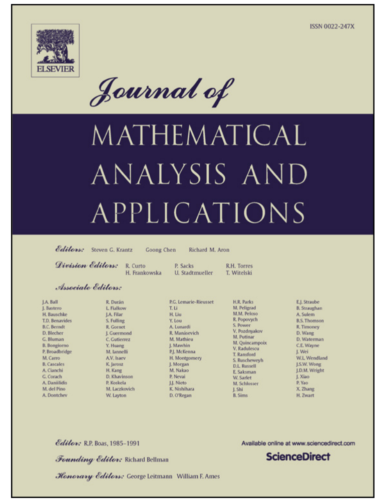

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A Unified Class of Integral Transforms Related to the Dunkl Transform

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Abstract

In the present paper, a new family of integral transforms depending on two parameters and related to the Dunkl transform is introduced. Well-known transforms, such as the fractional Dunkl transform, Dunkl transform, linear canonical transform, canonical Hankel transform, Fresnel transform etc, can be seen to be special cases of this general transform. Some useful properties of the considered transform such as Riemann-Lebesgue lemma, reversibility property, additivity property, operational formula, Plancherel formula, Bochner type identity and master formula are derived. The intimate connection that exists between this transformation and the quantum harmonic oscillator is developed.

Keywords: Canonical commutation relation, Dunkl transform, fractional Dunkl transform, Generalized Hermite polynomials and functions, semigroups of operators.

1 Intorduction

Integral transforms provide effective ways to solve a variety of problems arising in pure and applied mathematics. One example is the linear canonical transform (LCT) which represents a class of integral transforms indexed by a matrix parameter $M \in SL(2,\mathbb{R})$ [4]. Many well-known transforms such as Fourier transform, fractional Fourier transform, Weierstrass transform and Fresnel transform can be considred as special cases of this transformation (see [4, 32, 33]). While the theory of classical Fourier transform has a long and rich history, the growin interest in the theory of Dunkl transform, associated to a finite reflection groups and a multiplicity function k, is comparably recent. The Dunkl transform, which is a generalization of the Fourier and Hankel transforms, was introduced by C. F. Dunkl [9] and further studied by several authors (see [5, 9, 24]).

The primary aim of this article is to investigate a new integral transform that can unify all integral transforms stated in the previous paragraph. It seems desirable to have a more unified approach to all these integral transforms. According to literature M. Moshinsky and C. Quesne tackled this issue and considered that LCT is the group of unitary integral transforms that preserves the basic Heisenberg uncertainty relation of quantum mechanics in one or higher dimensions [22]. Furthermore, LCTs can be seen as the group of actions generated by the Lie algebra of quadratic Hamiltonian operators [32]. We briefly survey this mathematical framework. Let $\mathcal{H}$ be a Hilbert space. For a linear operator T on $\mathcal{H}$, we denote by $D(T)$ the domain of T. We say that a set $\{Q_j, P_j\}_{j=1}^N$ of self-adjoint operators on $\mathcal{H}$ is a representation of the canonical commutation relations (CCR) with N degrees of freedom [13], if there exists a dense subspace D of $\mathcal{H}$ such that
• $D \subset \cap_{j,k=1}^N [D(Q_jP_k) \cap D(P_kQ_j) \cap D(Q_jQ_k) \cap D(P_jP_k)]$
• Q_j and P_j satisfy on D the CCR

$$[Q_j, Q_k] = 0, \quad [P_j, P_k] = 0, \quad [Q_j, P_k] = \delta_k^j, \quad j, k = 1, \ldots, N,$$

where δ_k^j is the Kronecker symbol.

It is well known that a standard representation of the CCR is the Schrödinger representation $\left\{q_{\xi_j}, p_{\xi_j}\right\}_{j=1}^N$ which is given as follows: $\{\xi_j\}_{j=1}^N$ is an orthonormal basis of $\mathbb{R}^N$ with respect to the standard inner product $\langle . \rangle$, $\mathcal{H} = L^2(\mathbb{R}^N, dx)$, $q_{\xi_j} = \langle ., \xi_j \rangle$ (the multiplication operator by the jth coordinate $\langle x, \xi_j \rangle$), $p_{\xi_j} = \frac{1}{i}\frac{\partial}{\partial \xi_j}$ (

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