

ANALYTICITY OF SEMIGROUPS ON THE RIGHT HALF-PLANE

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ABSTRACT. This paper is devoted to the study of semigroups of composition operators and semigroups of holomorphic mappings. We establish conditions under which these semigroups can be extended in their parameter to a sector given a priori. We show that the size of this sector can be controlled by the image properties of the infinitesimal generator, or, equivalently, by the geometry of the so-called associated planar domain. We also give a complete characterization of all composition operators acting on the Hardy space H^p on the right half-plane.

Key words and phrases: holomorphic mapping, semigroup, composition operator, Hardy space, function convex in one direction.

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Dedicated to the 60-th birthday of Professor Dmitry Khavinson

1. INTRODUCTION

In this paper, we study semigroups of holomorphic self-mappings of the right half-plane $\Pi = \{z \in \mathbb{C} : \operatorname{Re} z > 0\}$ and semigroups of bounded linear operators acting on the Hardy spaces $H^p(\Pi)$, $p > 1$, and their interaction.

Recall that a one-parameter family $\mathbf{T} := \{T(t), t \geq 0\}$ of bounded linear operators on a Banach space X is said to be a C_0 -semigroup if it satisfies

- (a) $T(t)T(s) = T(t+s)$ for all $t, s \geq 0$,
- (b) $\lim_{t \rightarrow 0^+} T(t) = T(0) = I$, where I is the identity operator on X .

For each C_0 -semigroup \mathbf{T} on X , there exist scalars $a \geq 0$ and $M \geq 1$ such that $\|T(t)\| \leq Me^{at}$ for all $t \geq 0$. If $\|T(t)\| \leq 1$ for all $t \geq 0$, then the \mathbf{T} is said to be a contractive semigroup.

Denote $\mathcal{D} := \left\{ x \in X : \lim_{t \rightarrow 0} \frac{T(t)x - x}{t} \text{ exists} \right\}$. The linear operator Γ defined on \mathcal{D} by

$$\Gamma x := \lim_{t \rightarrow 0} \frac{T(t)x - x}{t} = \left. \frac{dT(t)x}{dt} \right|_{t=0}$$

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