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Numerical computation of connecting orbits in planar piecewise smooth dynamical system ☆

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A R T I C L E I N F O

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A B S T R A C T

In this paper, a numerical algorithm for computing the connecting orbits in piecewise smooth dynamical systems is derived and is analyzed. A nondegenerate condition for the connecting orbit with respect to its bifurcation parameter is presented to ensure the defining equation being well posed, which is a generalization of the Melnikov condition for smooth systems. The error caused by the truncation of time interval is also analyzed. Some numerical calculations are carried out to illustrate the theoretical analysis.

1. Introduction

In recent years, there are growing interests in piecewise smooth dynamical systems for their wide applications in applied science and engineering, such as the stick-slip mechanical systems [2,16,32], the mechanical systems with clearances or elastic constraints [34,33,36,37], the earthquake engineering [6,18,20], the power electronic converters [8,9,13], the suspension bridges [10] and so on. The discontinuity of the system is a special form of nonlinearity, which causes rich complicated new phenomena.

Our research interests in this work emanate from a model of a free-standing rigid block subjected to harmonic forcing, see Fig. 1 for a sketch. The model is often used to describe the behavior of man-made structures undergoing earthquakes.

The mathematical modeling of this rocking rigid block can be formulated as follows (see [21,18]),

$$\alpha\ddot{u} + \sin[\alpha(1-u)] = -\alpha\beta\cos[\alpha(1-u)]\cos\omega t, \qquad u > 0, \tag{1}$$

$$\alpha\ddot{u} - \sin[\alpha(1+u)] = -\alpha\beta\cos[\alpha(1+u)]\cos\omega t, \qquad u < 0, \tag{2}$$

$$\dot{u}(t^A) = r\dot{u}(t^B), \tag{3}$$

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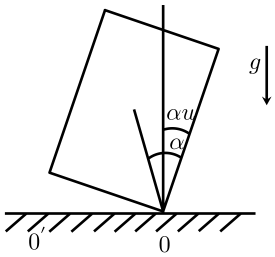

Fig. 1. Sketch of a rocking rigid block.

where α is the block shape parameter, αu presents the angle between one edge of the block and the droop, $\ddot{u}$ is the second derivative of u with respect to the time variable t, β and ω are amplitude and frequency parameters of the excitation, respectively. $0 \leqslant r \leqslant 1$ is the coefficient of restitution characterizing the energy loss at impact, t^A is the time just after impact and t^B is the time just before impact. If there is no external excitation the coefficient $\beta = 0$ and if the impact is completely elastic the parameter $r = 1$.

Much work has been carried out for the case $\alpha \ll 1$ (the slender block). In this situation, system (1)–(3) is reduced to a piecewise-linear system and its solutions can be obtained analytically. Hogan [19] shows that heteroclinic bifurcations appear in this piecewise smooth system. Bruhn and Koch [6] calculate a heteroclinic bifurcation condition without using perturbation methods and also use the Melnikov method in the case of small excitation and damping, etc. In many papers, they have mentioned that their methods also apply to the nonlinear case when α is an arbitrary angle which is related to the ordinary man-made structures.

One research interest on this problem is to study the structure stability of this rocking block. The terms in the right-hand side of equations (1)–(2) represent the external force added to the block undergoing earthquakes and equation (3) is the impact equation which reflects the ability of the block reverting to its original state during earthquakes. These two factors play an important role while studying the structure stability of the block. Besides these two external influences, the block's shape characterized by parameter α is also a key aspect to determine the stability of the block undergoing earthquakes. It is worthwhile to study the structure properties of the block without external influences, which correspond to studying the dynamics of equations (1)–(3) with parameters $\beta = 0$ and $r = 1$. It follows from numerical simulations that there exists a heteroclinic loop in the phase space $(u, \dot{u})$, inside of which are piecewise smooth periodic solutions corresponding to bounded oscillations of the block around the rest situation $u = \dot{u} = 0$, outside of which are orbits of large scale motions leading to overturning. This heteroclinic loop is the separatrix to distinct the stable and unstable motions of the block. In other words, it characterizes the critical situation of the block changing from stable motions to unstable motions. The main purpose of this paper is to construct a numerical method for computing the connecting orbits including homoclinic and heteroclinic orbits in planar piecewise smooth dynamical systems.

The numerical methods for computing connecting orbits in smooth dynamical systems are well studied by many authors, see [4,14,15,26] and the references therein. But these well posed methods are unable to be directly applied to piecewise smooth dynamical systems due to the influences by the discontinuity of the system. In this paper we study the numerical method for approximating a connecting orbit which transversally intersects the line of discontinuity. We define a nondegenerate condition for the piecewise smooth connecting orbit together with its bifurcation parameter, which is the generalization of the counterpart in smooth dynamical systems. In the geometric point of view, this nondegenerate condition is interpreted as that the stable and unstable manifolds pass each other along the line of discontinuity with non-vanishing velocity with respect to the bifurcation parameter. This nondegenerate condition ensures the regularity of an extending equation for computing a piecewise smooth connecting orbit together with its bifurcation parameter.