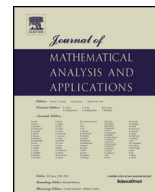




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Supercongruences on some binomial sums involving Lucas sequences [☆]

Guo-Shuai Mao, Hao Pan ^{*}

Department of Mathematics, Nanjing University, Nanjing 210093, People's Republic of China

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ABSTRACT

In this paper, we confirm several conjectured congruences of Sun concerning the divisibility of binomial sums. For example, with help of a quadratic hypergeometric transformation, we prove that

$$\sum_{k=0}^{p-1} \binom{p-1}{k} \binom{2k}{k}^2 \frac{P_k}{8^k} \equiv 0 \pmod{p^2}$$

for any prime $p \equiv 7 \pmod{8}$, where P_k is the k -th Pell number. Further, we also propose three new congruences of the same type.

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1. Introduction

In [10], with help of the Gross–Koblitz formula, Mortenson solved a conjecture of Rodriguez-Villegas [16] as follows:

$$\sum_{k=0}^{p-1} \binom{2k}{k}^2 \frac{1}{16^k} \equiv \left(\frac{-1}{p} \right) \pmod{p^2}$$

for every odd prime p , where $\left(\frac{\cdot}{p} \right)$ denotes the Legendre symbol. Subsequently, the similar congruences were widely studied. For the progress of this topic, the reader may refer to [11,12,14,6,17–19,8,20,7,4,21]. In [22], Sun proposed many conjectured congruences on the sums of binomial coefficients. Some of those conjectures are of the form

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^{*} Corresponding author.

E-mail addresses: mg1421007@smail.nju.edu.cn (G.-S. Mao), haopan79@zoho.com (H. Pan).

$$\sum_{k=0}^{p-1} \binom{p-1}{k} \binom{2k}{k}^2 a_n \equiv 0 \pmod{p^2}.$$

For example, Sun conjectured that

$$\sum_{k=0}^{p-1} \binom{p-1}{k} \binom{2k}{k}^2 \frac{\chi_3(k)}{16^k} \equiv 0 \pmod{p^2}$$

for any prime $p \equiv 1 \pmod{12}$, where $\chi_3(k)$ equals to the Legendre symbol $\left(\frac{k}{3}\right)$.

The main purpose of this paper is to confirm the following conjectures of Sun.

Theorem 1.1. *Suppose that p is a prime.*

(i) *If $p \equiv 3 \pmod{4}$, then*

$$\sum_{k=0}^{p-1} \binom{p-1}{k} \binom{2k}{k}^2 \frac{1}{(-8)^k} \equiv 0 \pmod{p^2}. \quad (1.1)$$

(ii) *If $p \equiv 1 \pmod{12}$, then*

$$\sum_{k=0}^{p-1} \binom{p-1}{k} \binom{2k}{k}^2 \frac{\chi_3(k)}{16^k} \equiv 0 \pmod{p^2}. \quad (1.2)$$

(iii) *If $p \equiv 7 \pmod{8}$, then*

$$\sum_{k=0}^{p-1} \binom{p-1}{k} \binom{2k}{k}^2 \frac{P_k}{8^k} \equiv 0 \pmod{p^2}, \quad (1.3)$$

where the Pell number P_k is given by

$$P_0 = 0, \quad P_1 = 1, \quad P_n = 2P_{n-1} + P_{n-2} \text{ for } n \geq 2.$$

(iv) *If $p \equiv 11 \pmod{12}$, then*

$$\sum_{k=0}^{p-1} \binom{p-1}{k} \frac{R_k}{(-4)^k} \binom{2k}{k}^2 \equiv 0 \pmod{p^2}, \quad (1.4)$$

where R_k is given by

$$R_0 = 2, \quad R_1 = 4, \quad R_n = 4R_{n-1} - R_{n-2} \text{ for } n \geq 2.$$

We mention that (1.1), (1.2), (1.3) and (1.4) respectively belong to Conjecture 5.5 of [18] and Conjectures A56, A57, A63 of [22].

The sequences $\{P_n\}$ and $\{R_n\}$ in Theorem 1.1 both belong to the second-order linear recurrence sequence. In general, define the Lucas sequences $\{U_n(a, b)\}$ and $\{V_n(a, b)\}$ by

$$U_0(a, b) = 0, \quad U_1(a, b) = 1, \quad U_n(a, b) = aU_{n-1}(a, b) - bU_{n-2}(a, b) \text{ for } n \geq 2,$$

and

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