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Relaxation limit in bipolar semiconductor hydrodynamic model with non-constant doping profile

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ABSTRACT

The relaxation limit from bipolar semiconductor hydrodynamic (HD) model to drift–diffusion (DD) model is shown under the non-constant doping profile assumption for both stationary solutions and global-in-time solutions, which satisfy the general form of the Ohmic contact boundary condition. The initial layer phenomenon will be analyzed because the initial data is not necessarily in the momentum equilibrium. Due to the bipolar coupling structure, the analysis is hard and different from the previous literature on unipolar model or bipolar model with zero doping profile restriction. We first construct the non-constant uniform stationary solutions by the operator method for both HD and DD models in a unified procedure. Then we prove the global existence of DD model and uniform global existence of HD model by the elementary energy method but with some new developments. Based on the above existence results, we further calculate the convergence rates in relaxation limits.

1. Introduction

We consider the following bipolar isothermal hydrodynamic (HD) model for semiconductors

$$\begin{cases} n_{it} + j_{ix} = 0, & \text{(a)} \\ j_{it} + \left(j_i^2/n_i + K_i n_i\right)_x = (-1)^{i-1} n_i \phi_x - j_i/\tau, & \text{(b)} \\ \phi_{xx} = n_1 - n_2 - D(x), \quad i = 1, 2, \quad \forall (t,x) \in (0,+\infty) \times \Omega, & \text{(c)} \end{cases} \tag{1.1}$$

where $\Omega := (0, 1)$ is a bounded interval occupied by the semiconductor device. The unknown functions $n_i(t, x)$ and $j_i(t, x)$ stand for the charge density, current distribution for electrons ($i = 1$) and holes ($i = 2$)

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respectively, and ϕ is the electrostatic potential. The positive constants τ, K_1 and K_2 are the relaxation time, temperature constant of electrons and temperature constant of holes respectively. The given function $D(x)$ means the non-constant doping profile, the density of impurities in semiconductor devices. Mathematically, the system (1.1) takes the form of the compressible fluids coupled with self-consistent Poisson equation, which leads to a hyperbolic–elliptic system.

In the present paper, we are interested in the behavior of solutions of the bipolar HD model (1.1) as the relaxation time $\tau \to 0^+$. Thus, we suppose $\tau \in (0, 1]$ and introduce a scaling of time $s = \tau t$ and define

$$n_i^\tau(s,x) = n_i\left(\frac{s}{\tau},x\right), \quad j_i^\tau(s,x) = \frac{1}{\tau}j_i\left(\frac{s}{\tau},x\right), \quad \phi^\tau(s,x) = \phi\left(\frac{s}{\tau},x\right). \tag{1.2}$$

Substituting the scaling transform (1.2) into the original HD model (1.1) and setting again $t = s$, we obtain the scaled HD model

$$\begin{cases} n_{it}^\tau + j_{ix}^\tau = 0, & \text{(a)} \\ \tau^2 j_{it}^\tau + \left(\tau^2 (j_i^\tau)^2/n_i^\tau + K_i n_i^\tau\right)_x = (-1)^{i-1} n_i^\tau \phi_x^\tau - j_i^\tau, & \text{(b)} \\ \phi_{xx}^\tau = n_1^\tau - n_2^\tau - D(x), \quad i = 1,2, \quad \forall (t,x) \in (0,+\infty)\times\Omega. & \text{(c)} \end{cases} \tag{1.3}$$

From now on, we only consider the scaled HD model (1.3) and also call it the HD model. The system (1.3) is complemented by the initial and boundary data

$$(n_i^\tau, j_i^\tau)(0,x) = (n_{i0}, j_{i0})(x), \tag{1.4}$$

and

$$n_i^\tau(t,0) = n_{il} > 0, \qquad n_i^\tau(t,1) = n_{ir} > 0, \tag{1.5a}$$

$$\phi^\tau(t,0) = 0, \qquad \phi^\tau(t,1) = \phi_r > 0, \tag{1.5b}$$

where n_{il}, n_{ir} and ϕ_r are positive constants. The physical boundary condition (1.5) is called the Ohmic contact boundary condition. Since we intend to establish the existence of a classical solution to the initial–boundary value problem (IBVP for abbreviation) (1.3)–(1.5), it is necessary to assume that the initial data (1.4) are compatible with the boundary data (1.5). Namely,

$$n_{i0}(0) = n_{il}, \quad n_{i0}(1) = n_{ir}, \quad j_{i0x}(0) = j_{i0x}(1) = 0. \tag{1.6}$$

Formally substituting $\tau = 0$ into the HD model (1.3) and expressing the solution of the limit system by $(n_1^0, j_1^0, n_2^0, j_2^0, \phi^0)$, we have the bipolar drift–diffusion (DD) model

$$\begin{cases} n_{it}^0 + j_{ix}^0 = 0, & \text{(a)} \\ j_i^0 = (-1)^{i-1} n_i^0 \phi_x^0 - K_i n_{ix}^0, & \text{(b)} \\ \phi_{xx}^0 = n_1^0 - n_2^0 - D(x), \quad i = 1,2, \quad \forall (t,x) \in (0,+\infty)\times\Omega. & \text{(c)} \end{cases} \tag{1.7}$$

The initial and boundary data for the DD model (1.7) are given by

$$n_i^0(0,x) = n_{i0}(x), \tag{1.8}$$

and