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Stability of Mindlin–Timoshenko plate with nonlinear boundary damping and boundary sources

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ABSTRACT

Presented here is a study of long-term behavior of Mindlin–Timoshenko (RMT) plate systems, focusing on the interplay between nonlinear viscous boundary damping and boundary source terms. This work complements [28] which established local well-posedness of this problem, and global well-posedness when the boundary damping dominates the boundary sources (in an appropriate sense). The current paper develops the potential well theory for the RMT system: global existence for potential well solutions without restricting the boundary source exponents, and explicit energy decay rates dependent on the boundary damping exponents. This work along with [26–28] provides the fundamental well-posedness and stability theory for MT plates under the interplay of damping and source terms acting either in the interior or on the boundary of the plate.

1. Introduction

The system of equations proposed by R. Mindlin [19] for describing mechanical vibrations stands among the most popular fundamental plate theory models. It can be viewed as a generalization of the Timoshenko beam to thin plates; also a similar model had been developed independently by E. Reissner. Thus, historically this framework could be referenced as Reissner–Mindlin–Timoshenko (RMT) plate theory:

$$\begin{cases} \rho h w_{tt} - K\Delta w - K(\psi_x + \phi_y) = 0, \\ \dfrac{\rho h^3}{12}\psi_{tt} - D(\psi_{xx} + \dfrac{1-\mu}{2}\psi_{yy}) - D\dfrac{1+\mu}{2}\phi_{xy} + K(\psi + w_x) = 0, \\ \dfrac{\rho h^3}{12}\phi_{tt} - D(\dfrac{1-\mu}{2}\phi_{xx} + \phi_{yy}) - D\dfrac{1+\mu}{2}\psi_{xy} + K(\phi + w_y) = 0, \end{cases} \tag{RMT}$$

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- Ω: mid-surface of the plate in equilibrium, occupying a subset of the (x, y)-plane;
- w: point-wise vertical displacement of the mid-surface;
- ψ: rotation angle of the filaments in the xz-plane;
- ϕ: rotation angle of the filaments in the yz-plane;
- h is the (uniform) plate thickness;
- ρ is the (constant) mass density per unit of surface area;
- $D = \dfrac{Eh^3}{12(1-\mu^2)}$ is the modulus of flexural rigidity;
- E is Young's modulus;
- μ is Poisson's ratio ($0 < \mu < 1/2$ in physical situation);
- $K = \dfrac{kEh}{2(1+\mu)}$ is the shear modulus; the factor k here is called the shear correction coefficient, it is introduced to account for the fact that the shearing strains are not uniform over a cross section of the plate.

RMT equations refine the classical Kirchhoff–Love model by taking into account shear deformations and thus relax the assumption that the filaments of the plate must remain perpendicular to its mid-plane. This formulation is substantially more accurate at high frequencies and when describing thicker plates, the RMT model has attracted a lot of research efforts.

There is an ample collection of work on the subject devoted to theoretic developments and numerical analysis, see for instance [2,7–10,13,21,22,24,25,31] and many references therein (for more detailed information, refer, for instance to the introductory sections in [26–28]).

The discussion of nonlinear damping-source interactions for a wave equation is initiated by Lasiecka and Tataru [14] and by Georgiev and Todorova [6], however, there has been less focus on the interaction of nonlinear sources and damping terms within the RMT framework. In 2015, [26,27] studied well-posedness and stability of RMT plate systems, focusing on the interaction between nonlinear interior damping and interior source terms.

This article complements [28] which previously addressed: local well-posedness of the system with boundary damping and boundary sources and global well-posedness when damping exponents dominate those of the sources. The present article:

- develops the potential well theory for the RMT model, proving global well-posedness for appropriate potential well solutions in presence of unrestricted boundary sources,
- establishes asymptotic uniform energy decay rates and their dependence on the damping nonlinearities.

1.1. The model

In the MT model the state of the system is represented by a vector-valued function $u = (w, \psi, \phi)$ which depends on position vector $\mathbf{x} = (x, y)$ and time $t \geq 0$. The component $w = w(\mathbf{x}, t)$ corresponds to the vertical displacement of the plate's mid-surface at point $\mathbf{x}$ time t, whereas ψ and ϕ are proportional to the angles of the plate filaments transversal to the mid-surface. Throughout the paper we assume that the mid-surface of the plate $\Omega \subset \mathbb{R}^2$ is a bounded open domain of class C^2 (hence has uniform C^2 regularity property, see [28] for detailed explanation of boundary regularity). The MT system reads:

$$\begin{cases} u_{tt} - \operatorname{div} S(u) + Q(u) = 0 & \text{in} \quad \Omega \times (0,T), \\ S_\nu(u) + u + (\mathscr{G}(u_t)) = (\mathscr{F}(u)) & \text{on} \quad \Gamma \times (0,T), \end{cases} \tag{1.1}$$

with Cauchy data prescribed in the *finite energy space* $(H^1(\Omega))^3 \times (L^2(\Omega))^3$, i.e.,