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Asymptotic behavior of entire large solutions to semilinear elliptic equations

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A B S T R A C T

In this paper, by using Karamata regular variation theory, we study the exact asymptotic behavior of entire large solutions to $\Delta u = b(x)f(u)$, $x \in \mathbb{R}^N (N \geq 3)$, where $b \in \mathrm{C}(\mathbb{R}^\mathrm{N})$ is nonnegative and nontrivial in $\mathbb{R}^N$, $f \in C([0,\infty))$ is positive and non-decreasing on $(0,\infty)$, which satisfies Keller–Osserman condition and is rapidly varying or regularly varying at infinity with index $\gamma \geq 1$.

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1. Introduction and main results

The purpose of this paper is to investigate the exact asymptotic behavior of entire large solutions (or entire blow-up solutions) to the following elliptic equations

$$\Delta u = b(x)f(u), \tag{1.1}$$

where $x \in \mathbb{R}^N (N \geq 3)$, and an entire large solution (or entire blow-up solution) of Eq. (1.1) means that $u \in C^2(\mathbb{R}^N)$ solves Eq. (1.1) and $\lim_{|x|\to\infty} u(x) = \infty$. In this paper, we are supposing that f satisfies

($\mathbf{f_1}$) f is continuous and nondecreasing on $[0,\infty)$ with $f(0) = 0$ and $f(t) > 0$ if $t > 0$;
($\mathbf{f_2}$) the Keller–Osserman condition

$$\int_1^\infty (2F(t))^{-1/2} dt < \infty, \ F(t) = \int_0^t f(s) ds$$

holds.

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Moreover, we further assume that $f \in RV_\gamma$ $(\gamma \geq 1)$ or f satisfies the following condition

($\mathbf{f_3}$) $\lim_{t\to\infty}((F(t))^{1/2})' \int_t^\infty (F(s))^{-1/2} ds = 1$.

Let b satisfy

($\mathbf{b_1}$) $b \in C(\mathbb{R}^N)$ is nonnegative in $\mathbb{R}^N$;
($\mathbf{b_2}$) there exist $k \in \mathscr{K}$ and constant $\lambda \geq 2$ such that

$$0 < b_1 := \liminf_{|x|\to\infty} \frac{b(x)}{|x|^{-\lambda}k(|x|)} \leq b_2 := \limsup_{|x|\to\infty} \frac{b(x)}{|x|^{-\lambda}k(|x|)} < \infty$$

and

$$\int_{t_0}^{\infty} s^{1-\lambda} k(s) ds < \infty,$$

where $\mathscr{K}$ denotes the set of Karamata functions k defined on $[t_0, \infty)$ by

$$k(t) := c \exp\left(\int_{t_0}^{t} \frac{y(s)}{s} ds \right),\ t > t_0 > 0$$

with $c > 0$ and $y \in C([t_0, \infty))$ such that $\lim_{t\to\infty} y(t) = 0$.

The Eq. (1.1) arises from many branches of mathematics and applied mathematics, for instance, Riemannian geometry, applied statistics, mathematical physics and population dynamics and has been discussed extensively by many authors in different contexts.

Let Ω be a bounded domain with C^2-boundary in $\mathbb{R}^N$ $(N = 2)$. If $b \equiv 1$ in Ω, $f(u) = e^u$, Bieberbach [6] first studied the existence, uniqueness and asymptotic behavior of large solutions to Eq. (1.1), where "large solution" is understood as u solves Eq. (1.1) in Ω and

$$u|_{\partial\Omega} = \infty, \text{ i.e., } \lim_{x\to\partial\Omega} u(x) = \infty.$$

The above solutions are also called "boundary blow-up solutions". Then, Rademacher [39], using the ideas of Bieberbach, showed that the results still hold for $N = 3$. When $f(u) = e^u$ and b is continuous and strictly positive on $\bar{\Omega}$, Lazer and McKenna [28] extended the above results in a bounded domain Ω in $\mathbb{R}^N$ $(N \geq 1)$ with a uniform outer sphere condition. On the other hand, Keller [22] and Osserman [37] carried out a systematic research on Eq. (1.1) and obtain the following important results for the existence of classical large solutions:

(i) If $\Omega \subseteq \mathbb{R}^N$ is a bounded domain, $b \equiv 1$ on $\bar{\Omega}$ and f satisfies ($\mathbf{f_1}$), then Eq. (1.1) possesses a large solution $u \in C^2(\Omega)$ if and only if the Keller–Osserman condition ($\mathbf{f_2}$) holds.
(ii) If $b \equiv 1$ in $\mathbb{R}^N$, f satisfies ($\mathbf{f_1}$), then Eq. (1.1) possesses an entire large solution if and only if the condition ($\mathbf{f_2}$) false, i.e.,

$$\int_1^\infty (2F(t))^{-1/2} dt = \infty. \tag{1.2}$$

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