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Some results on function ${}_pR_q(\alpha, \beta; z)$

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we define the function ${}_pR_q(\alpha, \beta; z)$ and obtain its properties including analytic properties (type and order), contiguous relations, differential property and simple integrals. We also establish its relation with other well known special functions.

1. Introduction

The Gauss hypergeometric function [13] is defined as

$$ {}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}, \tag{1}$$

for $|z| < 1$, and c any nonnegative integer, where $(\gamma)_n$ is a Pochhammer symbol [13],

$$ (\gamma)_n = \gamma(\gamma+1)\ldots(\gamma+n-1) = \frac{\Gamma(\gamma+n)}{\Gamma(\gamma)}, n \geq 1, (\gamma)_0 = 1, \gamma \neq 0. \tag{2}$$

For $|z| < 1$, the generalized hypergeometric function [2] is defined by

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$$ {}_pF_q\left[\begin{matrix} a_1, a_2, \ldots, a_p \\ b_1, b_2, \ldots, b_q \end{matrix}\middle| z\right] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \ldots (a_p)_n}{(b_1)_n (b_2)_n \ldots (b_q)_n} \frac{z^n}{n!}, \tag{3} $$

where no denominator parameter is zero or nonnegative integer.

Wright [19] further extended the generalized hypergeometric function in the form of Fox-Wright function (also known as Fox-Wright Psi function or just Wright function) as

$$ {}_p\Psi_q(z) = \sum_{n=0}^{\infty} \frac{\Gamma(\alpha_1+\beta_1 n)\ldots\Gamma(\alpha_p+\beta_p n)}{\Gamma(p_1+\mu_1 n)\ldots\Gamma(p_q+\mu_q n)} \frac{z^n}{n!}, \tag{4} $$

where β_r and μ_t are real positive numbers such that

$$ 1 + \sum_{t=1}^{q} \mu_t - \sum_{r=1}^{p} \beta_r > 0. $$

When β_r and μ_t are equal to 1, Equation (4) differs from the generalized hypergeometric function $pFq(z)$ only by constant multiplier.

Sharma and Jain [16] extended the generalized hypergeometric function and named it generalized M-series. This is defined as

$$ \begin{aligned} & {}_p\overset{\alpha,\beta}{M}_q(a_1,\ldots,a_p; b_1,\ldots,b_q; z) := {}_p\overset{\alpha,\beta}{M}_q(z) \\ & = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \ldots (a_p)_k}{(b_1)_k (b_2)_k \ldots (b_q)_k} \frac{z^n}{\Gamma(\alpha k+\beta)}, \qquad z, \alpha, \beta \in C, \quad \mathrm{Re}(\alpha) > 0. \end{aligned} \tag{5} $$

The generalized M-series defined by (5) is the extension of both Mittag-Leffler function and generalized hypergeometric function. Also it is a special case of Wright generalized hypergeometric function (4).

The classical Mittag-Leffler function [10] was defined as

$$ E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+1)}, \tag{6} $$

where z is a complex variable and $\alpha \geq 0$, that occurs as the solution of fractional order differential equation or fractional order integral equations.

Wiman [18] suggested the generalization of $E_\alpha(z)$ as

$$ E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+\beta)}, \tag{7} $$

for $\alpha, \beta \in C$, $\mathrm{Re}(\alpha), \mathrm{Re}(\beta) > 0$, which is known as Wiman's function.

Prabhakar [11] further extended the Mittag-Leffler function as

$$ E^\gamma_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!}, \tag{8} $$

for $\alpha, \beta, \gamma \in C$, $\mathrm{Re}(\alpha), \mathrm{Re}(\beta), \mathrm{Re}(\gamma) > 0$.

Many other researcher such as Shukla and Prajapati [17], Salim [14] and Sharma [15] studied Mittag-Leffler function and gave its generalization.

Motivated by the aforesaid investigations, we define the function ${}_pR_q(\alpha, \beta; z)$, which is a generalization of the K-function [15] with modified convergence conditions. In Section 3, we establish its relation with