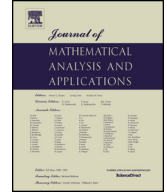




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Some results on function ${}_pR_q(\alpha, \beta; z)$

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ABSTRACT

In this paper, we define the function ${}_pR_q(\alpha, \beta; z)$ and obtain its properties including analytic properties (type and order), contiguous relations, differential property and simple integrals. We also establish its relation with other well known special functions.

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1. Introduction

The Gauss hypergeometric function [13] is defined as

$${}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}, \tag{1}$$

for $|z| < 1$, and c any nonnegative integer, where $(\gamma)_n$ is a Pochhammer symbol [13],

$$(\gamma)_n = \gamma(\gamma + 1) \dots (\gamma + n - 1) = \frac{\Gamma(\gamma + n)}{\Gamma(\gamma)}, n \geq 1, (\gamma)_0 = 1, \gamma \neq 0. \tag{2}$$

For $|z| < 1$, the generalized hypergeometric function [2] is defined by

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$${}_pF_q \left[\begin{matrix} a_1, a_2, \dots, a_p \\ b_1, b_2, \dots, b_q \end{matrix} \middle| z \right] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \dots (a_p)_n z^n}{(b_1)_n (b_2)_n \dots (b_q)_n n!}, \tag{3}$$

where no denominator parameter is zero or nonnegative integer.

Wright [19] further extended the generalized hypergeometric function in the form of Fox-Wright function (also known as Fox-Wright Psi function or just Wright function) as

$${}_p\Psi_q(z) = \sum_{n=0}^{\infty} \frac{\Gamma(\alpha_1 + \beta_1 n) \dots \Gamma(\alpha_p + \beta_p n) z^n}{\Gamma(p_1 + \mu_1 n) \dots \Gamma(p_q + \mu_q n) n!}, \tag{4}$$

where β_r and μ_t are real positive numbers such that

$$1 + \sum_{t=1}^q \mu_t - \sum_{r=1}^p \beta_r > 0.$$

When β_r and μ_t are equal to 1, Equation (4) differs from the generalized hypergeometric function ${}_pF_q(z)$ only by constant multiplier.

Sharma and Jain [16] extended the generalized hypergeometric function and named it generalized M-series. This is defined as

$$\begin{aligned} & {}_pM_q^{\alpha, \beta}(a_1, \dots, a_p; b_1, \dots, b_q; z) := {}_pM_q^{\alpha, \beta}(z) \\ & = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_p)_k z^k}{(b_1)_k (b_2)_k \dots (b_q)_k \Gamma(\alpha k + \beta)}, \quad z, \alpha, \beta \in C, \quad \text{Re}(\alpha) > 0. \end{aligned} \tag{5}$$

The generalized M-series defined by (5) is the extension of both Mittag-Leffler function and generalized hypergeometric function. Also it is a special case of Wright generalized hypergeometric function (4).

The classical Mittag-Leffler function [10] was defined as

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)}, \tag{6}$$

where z is a complex variable and $\alpha \geq 0$, that occurs as the solution of fractional order differential equation or fractional order integral equations.

Wiman [18] suggested the generalization of $E_{\alpha}(z)$ as

$$E_{\alpha, \beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)}, \tag{7}$$

for $\alpha, \beta \in C, \text{Re}(\alpha), \text{Re}(\beta) > 0$, which is known as Wiman’s function.

Prabhakar [11] further extended the Mittag-Leffler function as

$$E_{\alpha, \beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n z^n}{\Gamma(\alpha n + \beta) n!}, \tag{8}$$

for $\alpha, \beta, \gamma \in C, \text{Re}(\alpha), \text{Re}(\beta), \text{Re}(\gamma) > 0$.

Many other researcher such as Shukla and Prajapati [17], Salim [14] and Sharma [15] studied Mittag-Leffler function and gave its generalization.

Motivated by the aforesaid investigations, we define the function ${}_pR_q(\alpha, \beta; z)$, which is a generalization of the K-function [15] with modified convergence conditions. In Section 3, we establish its relation with

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