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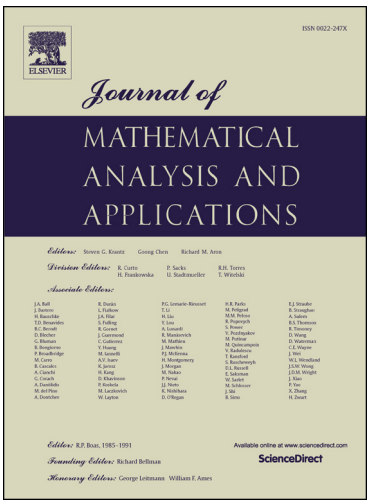

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A.F. Cheviakov, R. Naz

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A Recursion Formula for the Construction of Local Conservation Laws of Differential Equations

A. F. Cheviakov [a], R. Naz [b]

[a] *Department of Mathematics and Statistics, University of Saskatchewan, Saskatoon, Canada*

[b] *Centre for Mathematics and Statistical Sciences, Lahore School of Economics, Pakistan-53200, Lahore*

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Abstract

A simple formula is presented that, for any given local divergence-type conservation law of a system of partial or ordinary differential equations (PDE, ODE), generates a divergence expression involving an arbitrary function of all independent variables. In the cases when the new flux vector is a local expression inequivalent to the initial local conservation law flux vector, a new local conservation law is obtained. For ODEs, this can yield additional integrated factors. Examples of systems of differential equations are presented for which the proposed new relationship yields important local conservation laws starting from basic ones. Examples include a nonlinear ODE and several fundamental physical PDE models, in particular, general classes of nonlinear wave and diffusion equations, vorticity-type equations, and a shear wave propagation model in hyper-viscoelastic fiber-reinforced solids.

1 Introduction

Consider a system of differential equations (DE) $\mathcal{R}$ given by

$$R^\sigma[u] = 0, \qquad \sigma = 1, \dots, N, \tag{1.1}$$

with independent variable(s) $z = \{z^i\}_{i=1}^n$ and dependent variable(s) $u = \{u^k(z)\}_{k=1}^m$.

Definition 1. A *local divergence-type conservation law* of $\mathcal{R}$ is given by a divergence expression

$$\mathrm{D}_i \Psi^i[u] = 0 \tag{1.2}$$

that vanishes for all solutions $u(z)$ of the system (1.1).

In (1.2), $\Psi^i[u]$ are the conservation law *fluxes*, and

$$\mathrm{D}_i \equiv \mathrm{D}_{z^i} = \frac{\partial}{\partial z^i} + u^j_i \frac{\partial}{\partial u^j} + u^j_{ii_1} \frac{\partial}{\partial u^j_{i_1}} + u^j_{ii_1i_2} \frac{\partial}{\partial u^j_{i_1i_2}} + \cdots \tag{1.3}$$

[a)] Alternative English spelling: Alexey Shevyakov. Electronic mail: shevyakov@math.usask.ca

[b)] Electronic mail: drrehana@lahoreschool.edu.pk