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Delta shock wave in the compressible Euler equations for a Chaplygin gas ☆

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ABSTRACT

We study the Riemann problem of the compressible Euler equations for the Chaplygin gas. With the analysis on the physically relevant region, we obtain two kinds of Riemann solutions by using the method of characteristic analysis. One composes of three contact discontinuities, and the other involves a delta shock wave in which both density and internal energy contain Dirac delta function simultaneously. We propose both generalized Rankine–Hugoniot relation and entropy condition for this type of delta shock wave. The numerical results coinciding with the theoretical analysis are also presented.

1. Introduction

The compressible Euler equations read

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p(\rho, s))_x = 0, \\ (\rho u^2/2 + \rho e)_t + ((\rho u^2/2 + \rho e + p(\rho, s))u)_x = 0, \end{cases} \tag{1.1}$$

where the state variables ρ, u, s, p, e denote the density, the velocity, the specific entropy, the pressure and the specific energy. The p and e are given functions of ρ and s, governed the thermodynamical constraint

$$T\mathrm{d}s = \mathrm{d}e + p\,\mathrm{d}\tfrac{1}{\rho} \tag{1.2}$$

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with $T = T(\rho, s)$ being the temperature. Numerous excellent achievements on the Riemann problems of system (1.1) or other closely related problems have been made for the polytropic gas, see [3,12,24] and the references therein. Here, we concern with the equation of state

$$p(\rho, s) = -\tfrac{1}{\rho}, \tag{1.3}$$

which is called a Chaplygin gas. It was introduced by Chaplygin [4], Tsien [25] as a good mathematical approximation for calculating the lifting force on a wing of an airplane in aerodynamics. The Chaplygin gas owns a negative pressure and occurs in certain theories of cosmology. Such a gas was also advertised as a possible model of dark energy [1,9,20].

Many investigations have concentrated on the Chaplygin gas. Brenier [2] considered the Riemann problem for the isentropic Euler equations

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p(\rho, s))_x = 0, \end{cases} \tag{1.4}$$

where the solutions with concentration were obtained when initial data belong to a certain domain in the phase plane. Guo, Sheng and Zhang [10] abandoned this constraint and obtained all of the solutions in which the delta shock wave is developed. Besides, Wang [26], Yang and Wang [27], Sheng, Wang and Yin [23] studied further system (1.4) for the generalized Chaplygin gas, and the Riemann solutions and the limits of these solutions were analyzed. The Riemann problem for relativistic Chaplygin Euler equations was solved by Cheng and Yang [7]. Moreover, Zhu and Sheng [30] concerned with the Riemann problem for the compressible Euler equations (1.1), where the delta shock wave with Dirac delta function only in the density appeared in solutions. For the research on the Riemann problems of others one-dimensional or two-dimensional systems of conservation laws for the Chaplygin gas, we refer the readers to [5,10,17–19] and the references cited therein.

In 1979, Kraiko [11] considered system (1.1) with $p(\rho, s) = 0$, and the discontinuities which would be different from classical ones and carry mass, impulse and energy were needed to construct the solution for arbitrary initial data. Since both the density ρ and the specific energy e contain the Dirac delta function, it is impossible to define the product of them. Nilsson Rozanova and Shelkovich [15,16] denoted the internal energy ρe by a new variable H, and showed the processes of concentration of both mass and internal energy on the delta shock wave front. Cheng [6] considered the Riemann problem for (1.1) with $p(\rho, s) = 0$, where the delta shock wave with Dirac delta function in both the density and the internal energy developed in the solutions. For the theory of delta shock wave with Dirac delta function in multiple state variables, interested readers may refer to [8,21,22,28,29] for further details. Motivated by the idea in [15,16], in contrast to [30], we will consider the compressible Euler equations in the form

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p(\rho, s))_x = 0, \\ (\rho u^2/2 + H)_t + ((\rho u^2/2 + H + p(\rho, s))u)_x = 0, \end{cases} \tag{1.5}$$

where the state variable $H \geq 0$ is the internal energy.

In this paper, we study the Riemann problem of (1.5) and (1.3) with the initial data

$$(\rho, u, H)(0, x) = \begin{cases} (\rho_-, u_-, H_-), x < 0, \\ (\rho_+, u_+, H_+), x > 0, \end{cases} \tag{1.6}$$

where $\rho_i > 0$, $u_i, H_i > 0$, $i = -, +$, are different constants. According to the thermodynamical constraint (1.2), we first clarify the physically relevant region. With the help of the contact discontinuity