Doctopic: Applied Mathematics ARTICLE IN PRESS YJMAA:20884

J. Math. Anal. Appl. • • • (• • • •) • • •–• • •

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Approximate analytical solutions of nonlinear differential equations using the Least Squares Homotopy Perturbation Method

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A R T I C L E I N F O

Article history:
Received 9 July 2016
Available online xxxx
Submitted by S.A. Fulling

Keywords:
Least Squares Homotopy Perturbation Method
Nonlinear differential equations
Approximate analytical solution
Homotopy Perturbation Method

A B S T R A C T

In the present paper we introduce a new method to compute approximate analytical solutions for nonlinear differential equations, called the Least Squares Homotopy Perturbation Method. The method is based on the well-known Homotopy Perturbation Method and its main feature is an accelerated convergence compared to the regular Homotopy Perturbation Method. The comparison with previous results emphasizes the high accuracy of the method.

0. Nomenclature

Symbol	**Description**
HAM	Homotopy Analysis Method
HPM	Homotopy Perturbation Method
DTM	Differential Transform Method
MDTM	Multi-Step Differential Transform Method
$\mathfrak{L}$	Linear operator of the HPM
$\mathfrak{N}$	Nonlinear operator of the HPM
For Application 2	
Re	Reynolds number
Ha	Hartman number
α	Angle of the inclined plane walls of the channel

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http://dx.doi.org/10.1016/j.jmaa.2016.11.031

1. Introduction

Most of the phenomena encountered in nature and technology are modelled using nonlinear differential equations and, for this reason, it is very important to find solutions of these equations. In many cases, as the calculation of exact solutions is not possible, approximate solutions of the equations are computed.

Among the most well-known and most widely used methods to compute analytical approximate solutions for nonlinear differential equations we mention:

- The Homotopy Analysis Method (HAM), introduced by Liao [8] was one of the first analytic methods for nonlinear problems which avoids the need for a "small" parameter and thus is applicable to strong nonlinear problems. HAM has been successfully applied for many types of nonlinear problems from various fields of science and engineering such as, for example, [1,5,2].
- The Homotopy Perturbation Method (HPM), introduced by He [6,7] was also used to solve a wide range of strongly nonlinear problems (for example [12,9,11]).
- The Differential Transform Method (DTM), introduced by Zhou [13], is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. DTM together with its modifications (such as the multi-step DTM) has been also successfully employed to solve many types of nonlinear problems [2,10,4].

In the following we will introduce a new approximation method named Least Squares Homotopy Perturbation Method (LSHPM). As the name suggests, the method is based on the Homotopy Perturbation Method and its main feature is an accelerated convergence compared to the regular Homotopy Perturbation Method.

The examples presented show that the approximate solutions obtained by using LSHPM require less iterations in comparison with other iterative methods for approximate solutions of differential equations. Moreover, for an equal number of iterations, the approximate solutions obtained by using LSHPM are more accurate than the ones obtained by other methods.

2. The Least Squares Homotopy Perturbation Method

We consider the following problem, consisting of a differential equation and some boundary conditions:

$$\mathfrak{L}(x(t)) + \mathfrak{N}(x(t)) - f(t) = 0, \quad B(x) = 0 \tag{1}$$

Here $\mathfrak{L}$ is a linear operator, $x(t)$ is the unknown function, $\mathfrak{N}$ is a nonlinear operator, $f(t)$ is a known, given function and B is a boundary operator.

If $\tilde{x}$ is an approximate solution of equation (1), we evaluate the error obtained by replacing the exact solution x with the approximate one $\tilde{x}$ as the remainder:

$$R(t, \tilde{x}) = \mathfrak{L}(\tilde{x}(t)) + \mathfrak{N}(\tilde{x}(t)) - f(t), \quad t \in \mathbb{R} \tag{2}$$

The first step in applying LSHPM is to attach to the problem (1) the family of equations (see [6,7]):

$$(1-p)[\mathfrak{L}(\Phi(t,p)) - f(t)] + p\,[\mathfrak{L}(\Phi(t,p)) + \mathfrak{N}(\Phi(t,p)) - f(t)] = 0 \tag{3}$$

where $p \in [0,1]$ is an embedding parameter, $\Phi(t,p)$ is an unknown function.

When $p = 0$, $\Phi(t,0) = x_0(t)$ and when $p = 1$, $\Phi(t,1) = x(t)$. Thus, as p increases from 0 to 1, the solution $\Phi(t,p)$ varies from $x_0(t)$ to the solution $x(t)$, where $x_0(t)$ is obtained from the equation:

$$\mathfrak{L}(x_0(t)) - f(t) = 0, \quad B(x_0) = 0 \tag{4}$$

Please cite this article in press as: C. Bota, B. Căruntu, Approximate analytical solutions of nonlinear differential equations using the Least Squares Homotopy Perturbation Method, J. Math. Anal. Appl. (2017), http://dx.doi.org/10.1016/j.jmaa.2016.11.031