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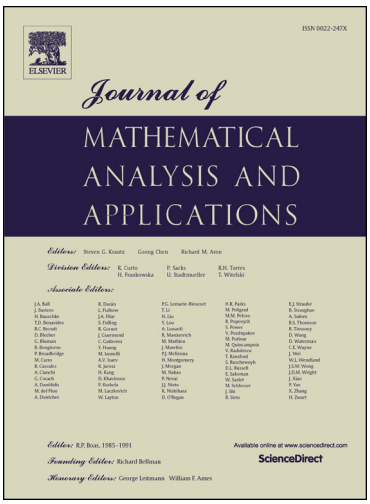

Qualitative properties of nonlinear parabolic operators

István Faragó, Róbert Horváth, János Karátson, Sergey Korotov

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Qualitative properties of nonlinear parabolic operators

István Faragó[b,c,*], Róbert Horváth[a,c], János Karátson[a,b,c,*], Sergey Korotov[d]

[a] *Department of Analysis, Budapest University of Technology and Economics, Budapest, Hungary*
[b] *Department of Applied Analysis and Computational Mathematics, ELTE University, Budapest, Hungary*
[c] *MTA-ELTE NumNet Research Group, Budapest, Hungary*
[d] *Bergen University College, Bergen, Norway*

Abstract

It is a natural expectation that the mathematical models of real-life phenomena have to possess some characteristic qualitative properties of the original process. For parabolic problems the main known qualitative properties are the maximum-minimum principles, nonnegativity-nonpositivity preservation and maximum norm contractivity. These properties have a fundamental relevance concerning the validity of the mathematical or numerical model: without them, the model might produce unphysical quantities that contradict reality. For linear problems with Dirichlet boundary conditions, these properties have been thoroughly investigated and their relations have been characterized. In the present paper, we extend the linear results to nonlinear problems with general boundary conditions. Firstly, we characterize various implications between the qualitative properties. Some of them are given in general, and in certain cases we restrict our study to operators with gradient-dependent principal part or to operators with heat conduction coefficient. Secondly, we give general sufficient conditions to ensure these qualitative properties, both separately and all of them together. The relations are illustrated with several examples.

Keywords: nonlinear parabolic problems, qualitative properties, maximum principle, numerical solution

1. Introduction

A large number of time-dependent real-life phenomena can be modelled mathematically by parabolic partial differential equations, such as heat conduction, diffusion, air pollution, option pricing, disease propagation [1, 3, 18, 21, 22, 24, 27], to name a few. The qualitative theory of partial differential

*Corresponding author
Email addresses: faragois@cs.elte.hu (István Faragó), rhorvath@math.bme.hu (Róbert Horváth), karatson@cs.elte.hu (János Karátson), sergey.korotov@hib.no (Sergey Korotov)